ETH zürich

P&S COMSOL® Design: Simulations of Optical Components Lecture 4: Wave Optics and Waveguiding

Taichiro Fukui, Maximilian Bosch

Content

- Last week
  - Young’s single/double slit experiment

- Today
  Review on material properties
  - Waveguide
    - Motivation
    - Theory
    - COMSOL

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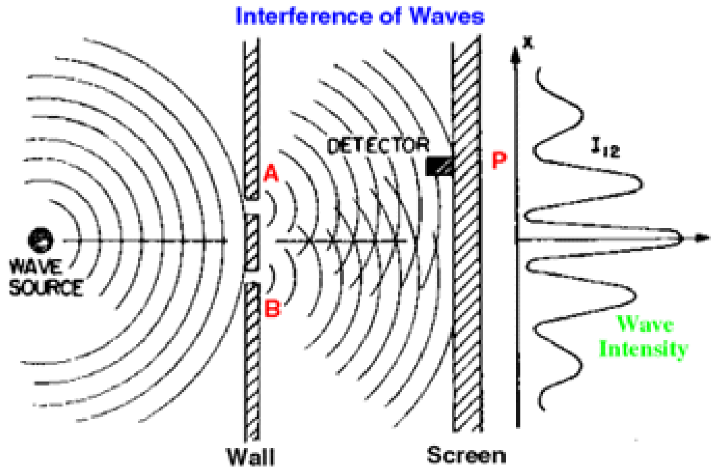
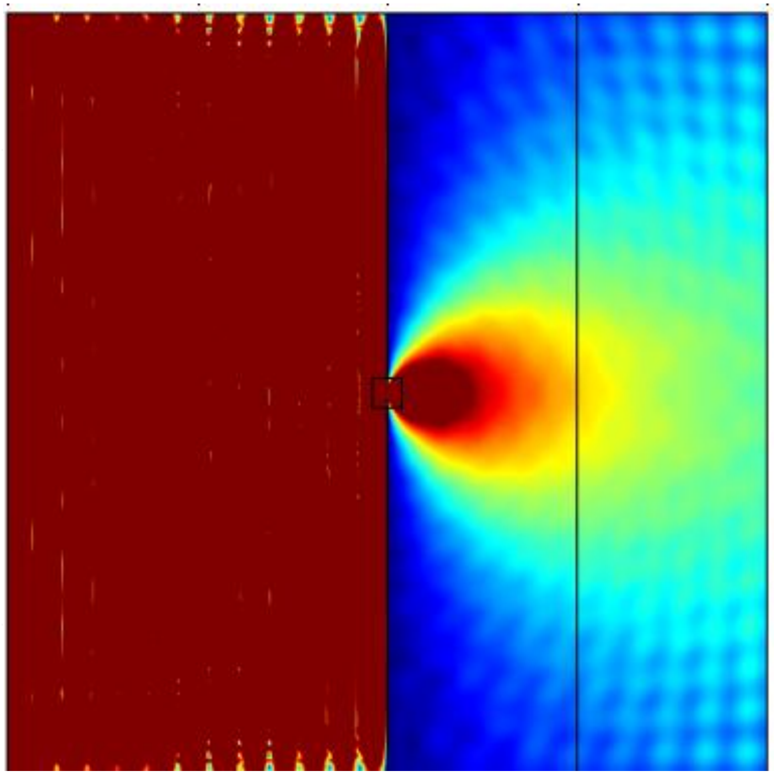
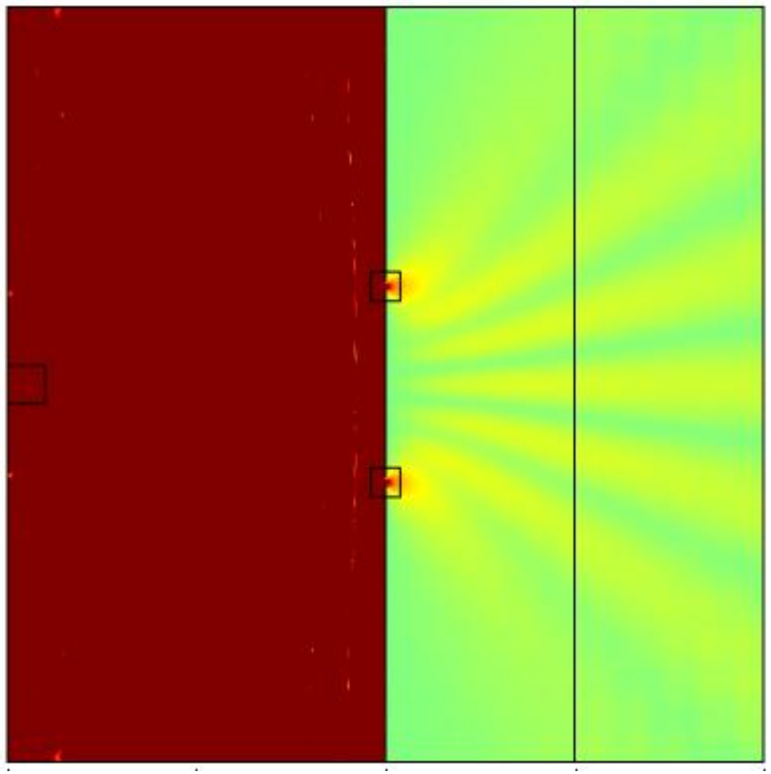
Last Week: Young's Slit Experiments



One Slit



Two Slits



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Review: Material Relations

- In order to analyze an EM problem we need to define the **material properties** involved
- Materials are defined by their **refractive index $\boldsymbol{n}$** which is defined as
  - $\boldsymbol{n} = \sqrt{\boldsymbol{\mu_r \varepsilon_r}}$, for vacuum $n = 1$
  - n is a complex number $n = n' + ik$

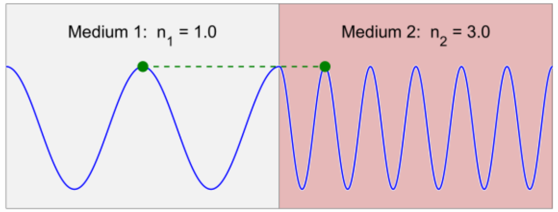
Influences wavelength (n')

Influences losses (k)

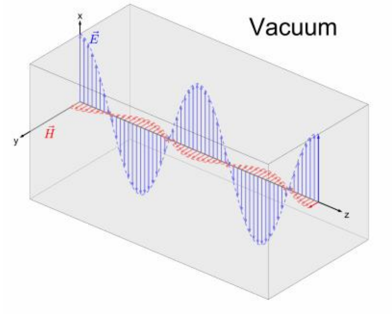
Antenna

Air

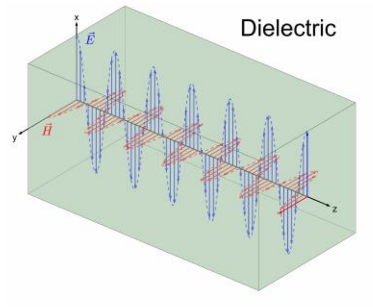
Glass



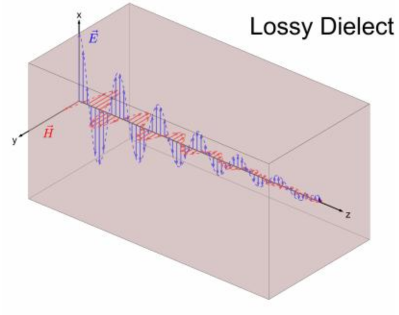
$n = 1$



$n = 3$



$n = 3 + 2i$



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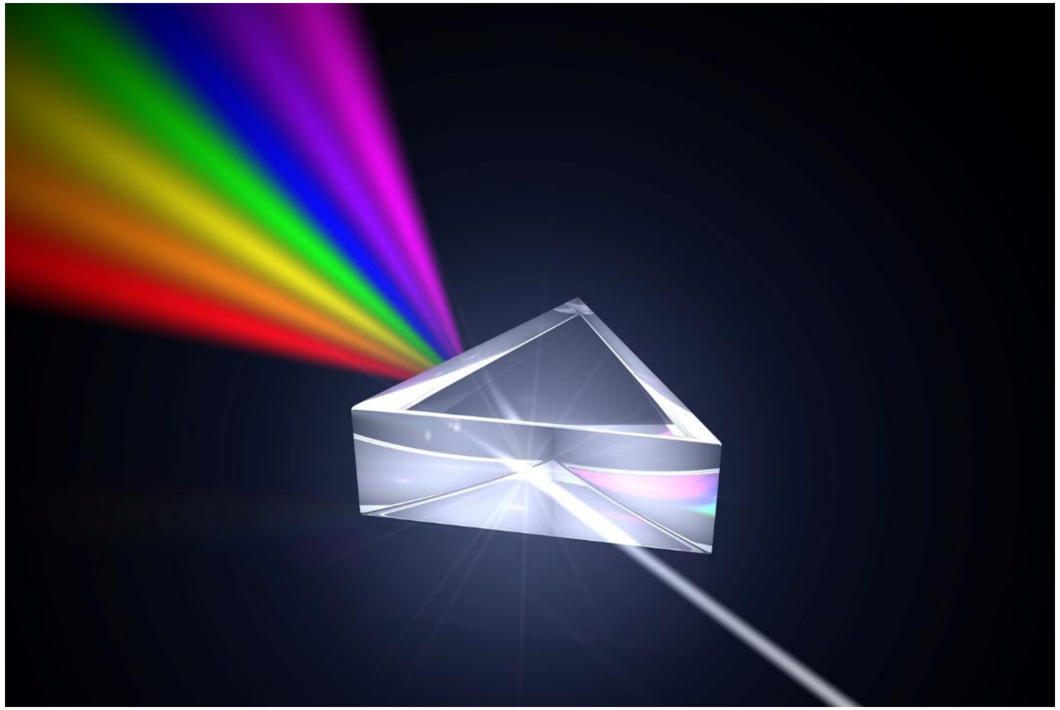
Material Properties

- What is the meaning of $n(\omega)$?
- Newton discovered that it changes with wavelength!
- This phenomena can be mathematically modelized



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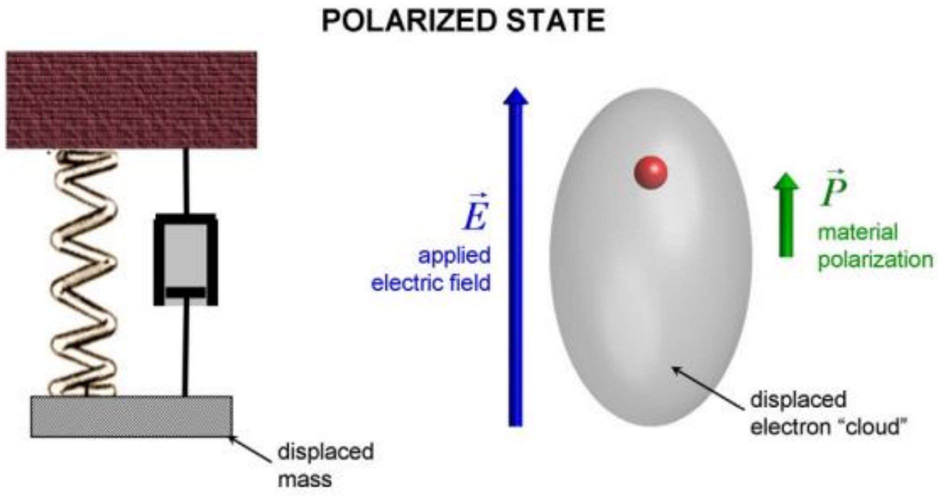
Material Properties

- What is the meaning of $n(\omega)$?
- Newton discovered that it changes with wavelength!
- This phenomena can be mathematically described using the **Lorentz Oscillator Model**

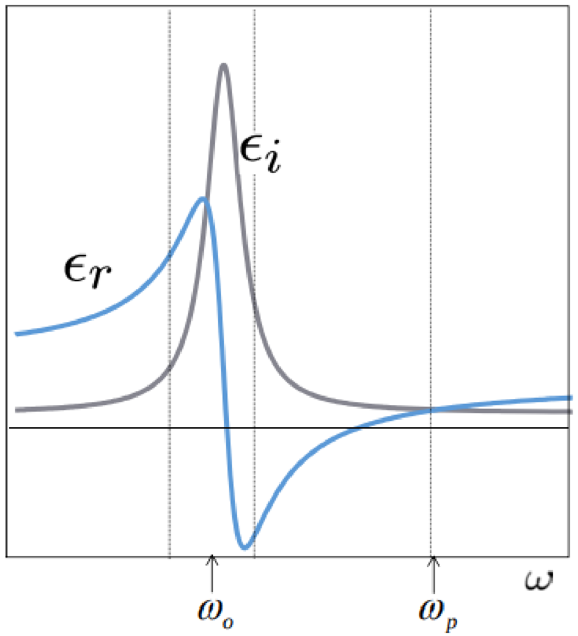


$$m\ddot{x} + \Gamma\dot{x} + kx = -e\boldsymbol{E} \leftrightarrow (j\omega)^2 x + j\omega x + kx = -e\boldsymbol{E}$$

$$\boldsymbol{P} = \frac{1}{V}\sum \boldsymbol{p_i} \; with \; \boldsymbol{p_i} = -ex(\omega) \; \boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E}(\omega)$$

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_0 \varepsilon_r \boldsymbol{P}$$

$$\varepsilon_r(\omega) = 1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 - j\omega\Gamma}$$



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Waveguiding: Motivation

- How does the internet work?

- Data transfer
  - Wireless
  - By cable

Electrical (copper) or optical (glass fiber)

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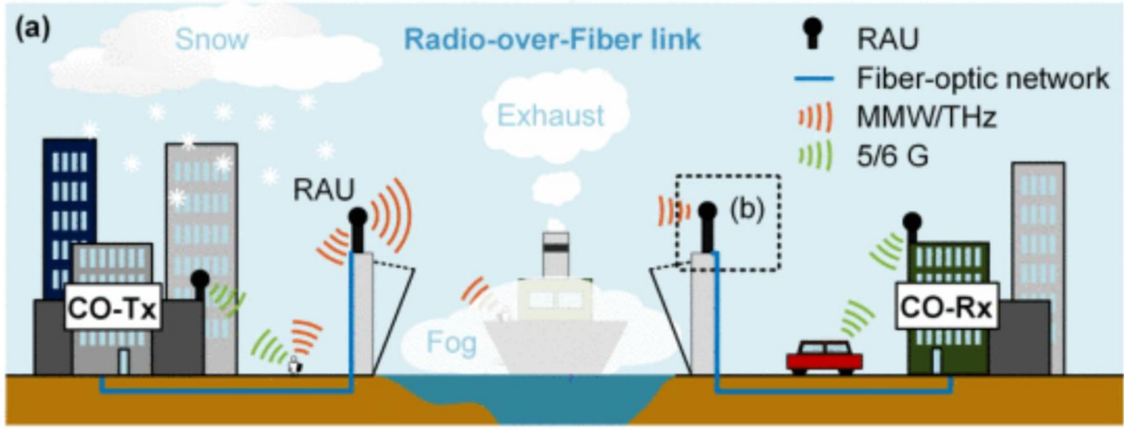
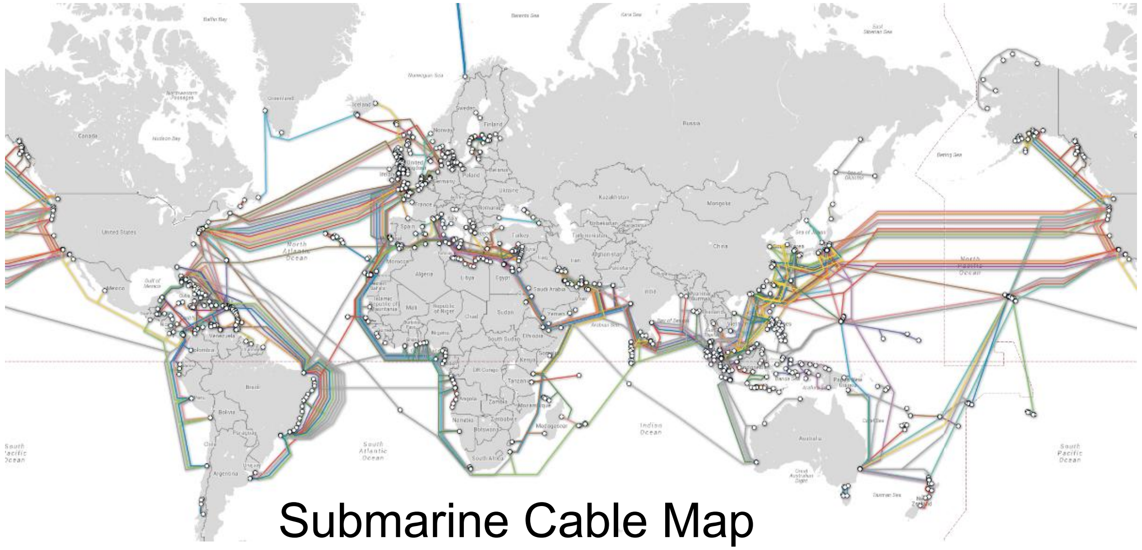
Waveguiding: Motivation

- Data transfer with an optical fiber





Submarine Cable Map

[1] B. Bitachon *et al.* , "Deep learning based digital backpropagation demonstrating SNR gain at low complexity in a 1200 km transmission link," *Opt. Express* 2020
[2] Y. Horst *et al*., "Transparent Optical-THz-Optical Link at 240/192 Gbit/s Over 5/115 m Enabled by Plasmonics," in *Journal of Lightwave Technology 2022*

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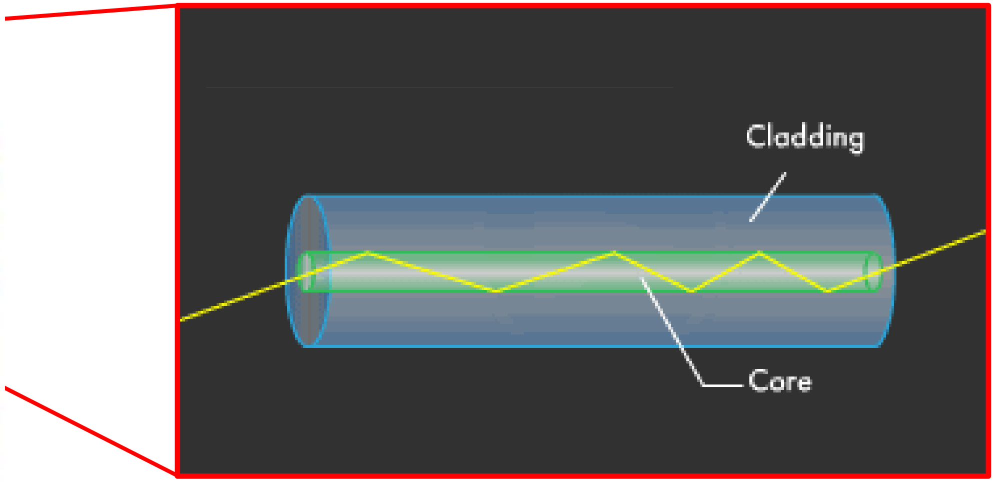
Waveguiding

- Optical data transmission → in the future maybe also in electronic devices (smartphones etc.)
- Our goal: learn how to simulate optical components used on-chip



How can we make this happen?

- Optical fiber often used terms:
  - Core
  - Cladding

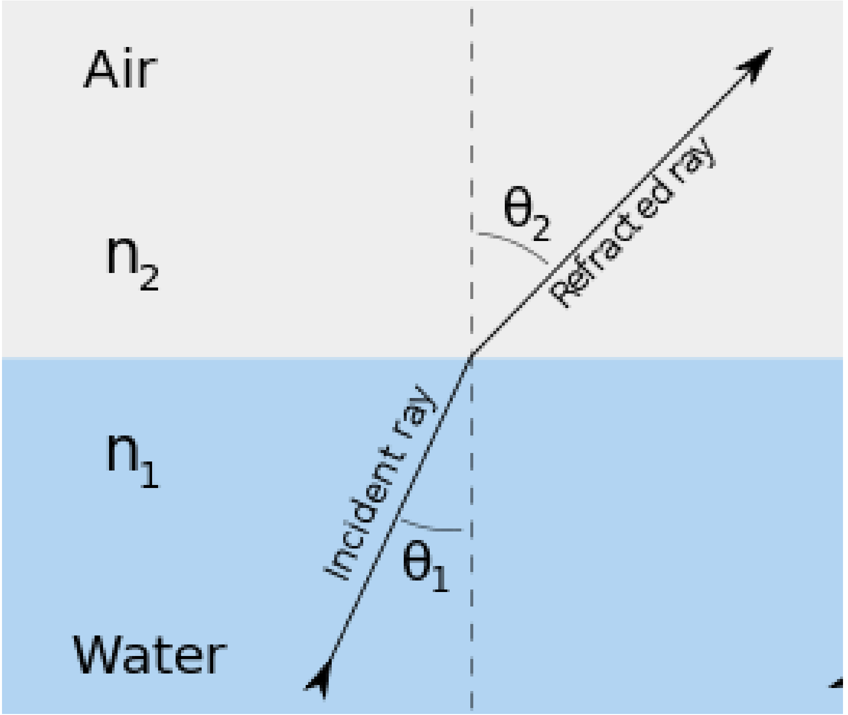
https://www.ednasia.com/a-new-step-towards-photonic-chips/

Waveguiding: Theory

- What happens at the boundary between two materials?

Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



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Waveguiding: Theory

Snell's law

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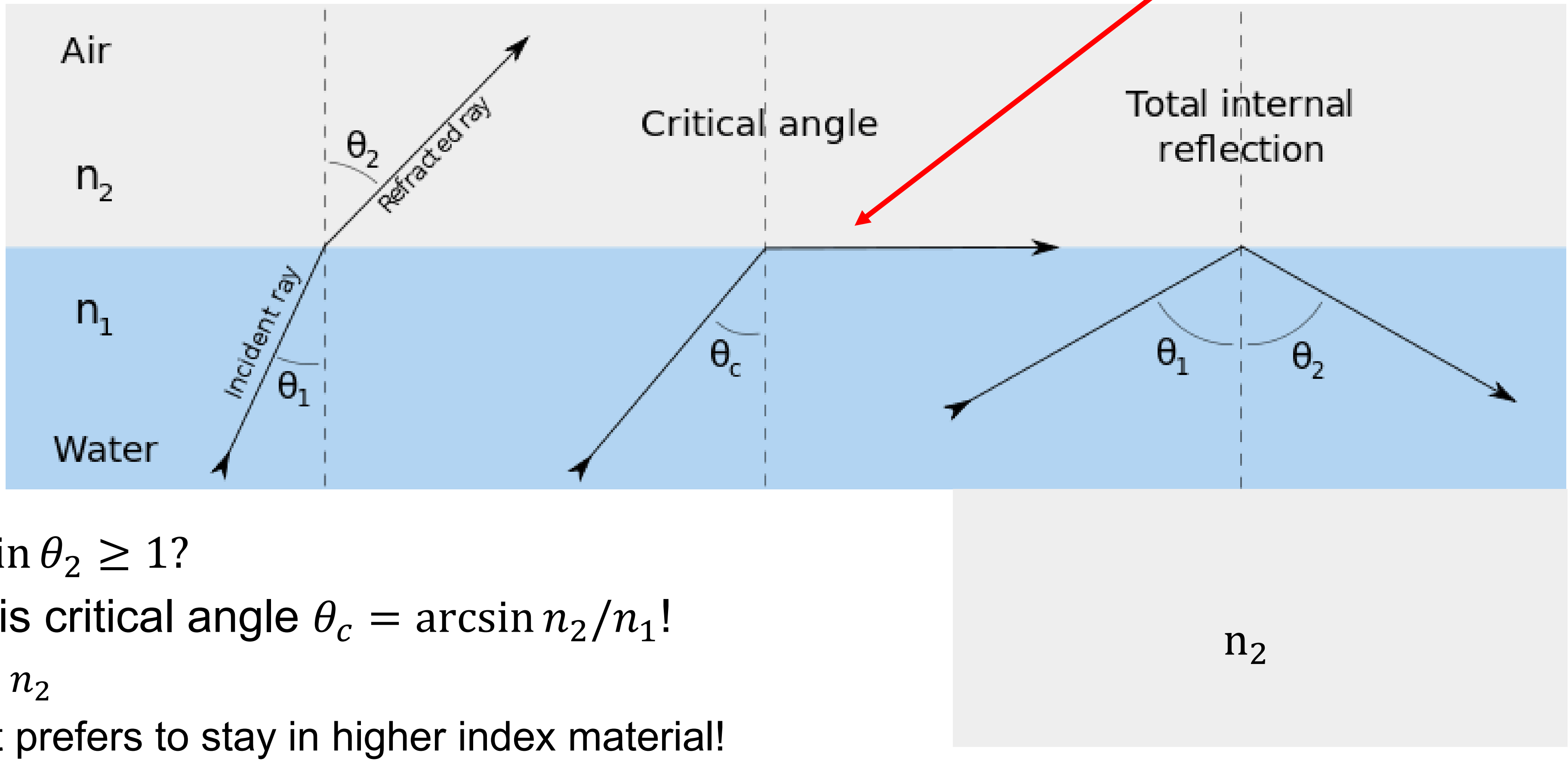
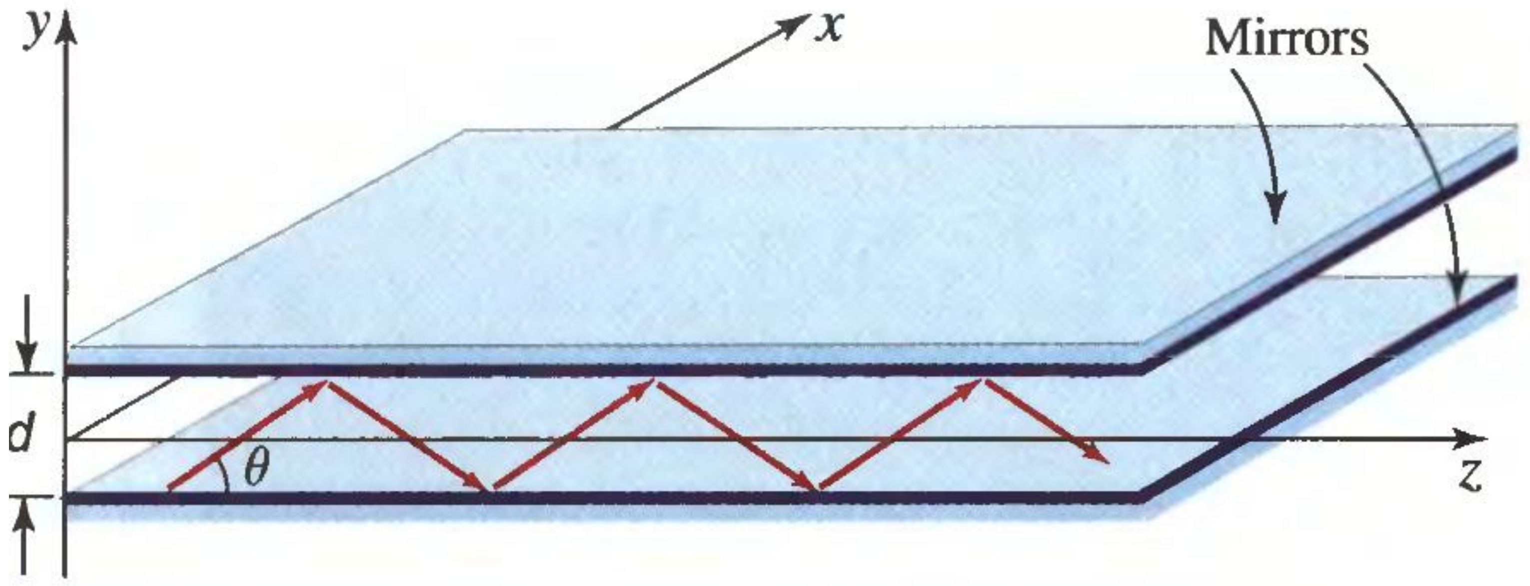


- What if $\sin\theta_2 \geq 1$?
  - There is critical angle $\theta_c = \arcsin n_2/n_1$!
    - $n_1 > n_2$
    - Light prefers to stay in higher index material!
    - Almost everything is reflected → **total internal reflection!**

Waveguiding: Theory

- EM field in between two perfect metallic mirrors
  - After each reflection, there is π phase shift



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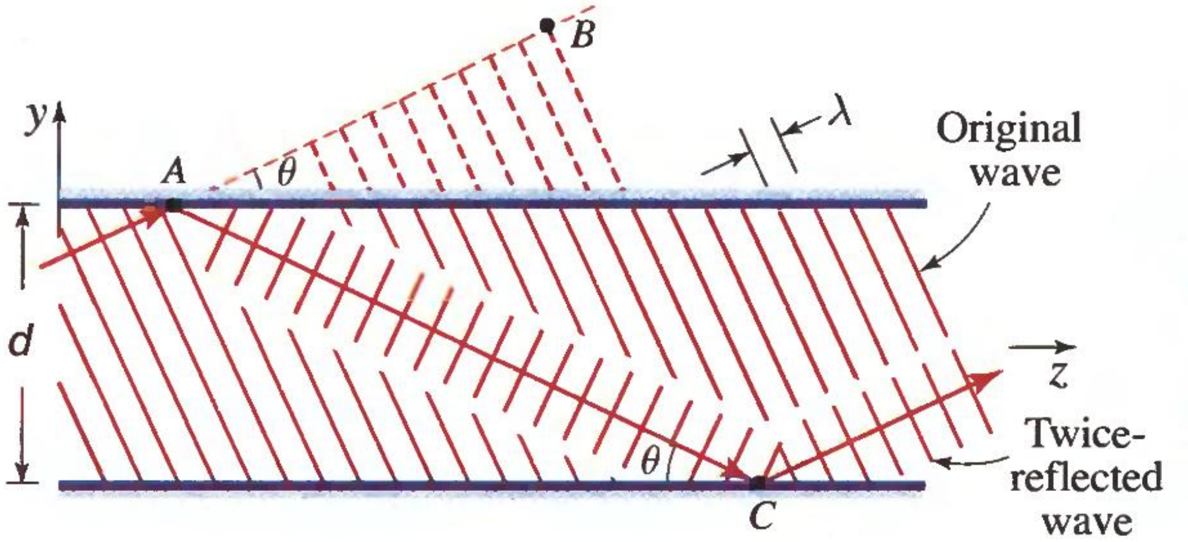
Waveguiding: Theory

- EM field in between two perfect mirrors
  - Interference after second reflection!
  - Self consistency: after second reflection, wave duplicates itself



$$\overline{AC} = \frac{d}{\sin(\theta)}, \overline{AB} = \overline{AC}\cos(2\theta)$$

$$\overline{AC} - \overline{AB} = 2d\sin(\theta)$$

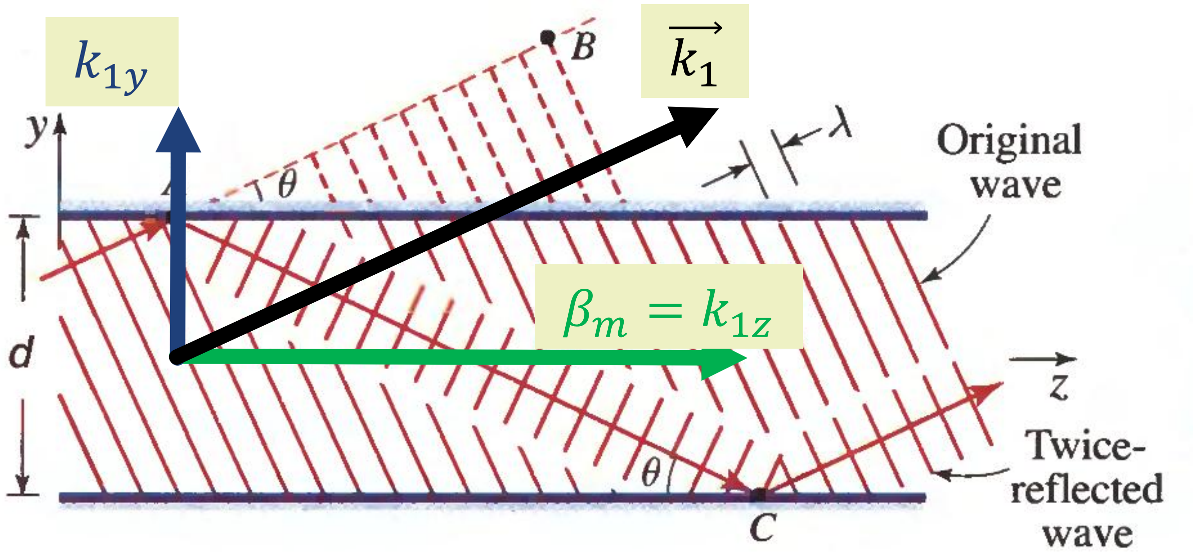
Constructive interference

$$\Delta\varphi = n_1 \frac{2\pi}{\lambda}(\overline{AC} - \overline{AB}) = 2\pi m$$

$$\sin(\theta_m) = \frac{\lambda m}{2d}$$

Waveguiding: Theory

- EM field in between two perfect mirrors
  - Interference after second reflection!
  - Self consistency: after second reflection, wave duplicates itself



$$k_{1y} = k_1 \sin\theta_m$$

$$k_{1z} = k_1 \cos\theta_m$$

with $k_1 = k_0 n_1$

Propagation constant

Effective refractive index

$$\beta_m = k_1 \cos\theta_m = k_1 \sqrt{1 - \left(\frac{2\lambda m}{d}\right)^2} = \frac{2\pi}{\lambda} n_{eff,m}$$

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Waveguiding: Theory

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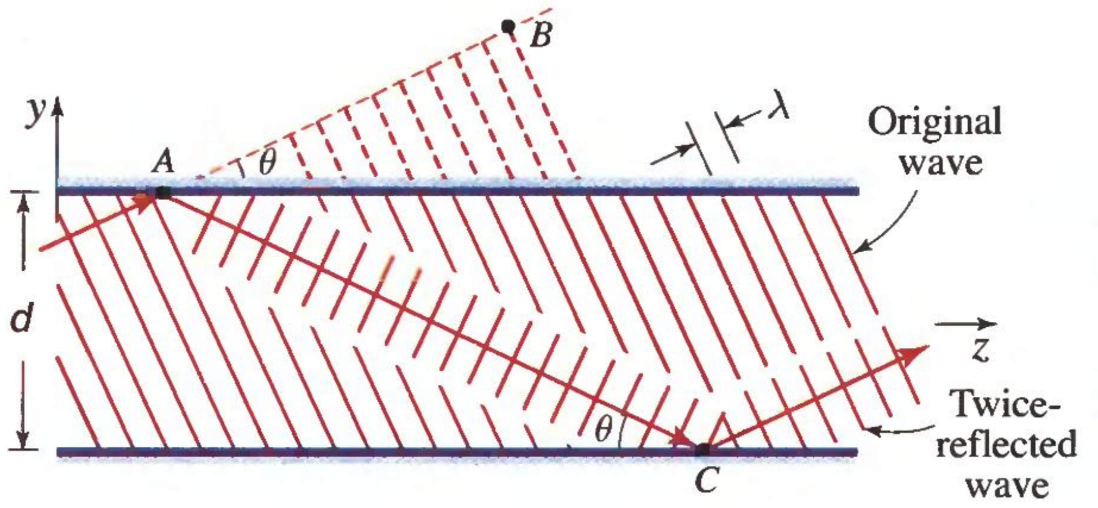

$$2d \cdot k_1 \sin\theta_m = 2\pi m$$

$$\sin\theta_m = \frac{\pi m}{dk_1} < 1$$

$$m < \frac{dk_1}{\pi} = \frac{2d}{\lambda} n$$

$$\omega_c \geq \pi \frac{c/n_1}{d}$$

Number of modes: $M < \frac{2d}{\lambda} n$

Cut-off frequency: $1 = \frac{2d}{\lambda_c} n_1 \Rightarrow \lambda_c = 2dn_1$

$$f_c = \frac{c}{2dn_1}$$

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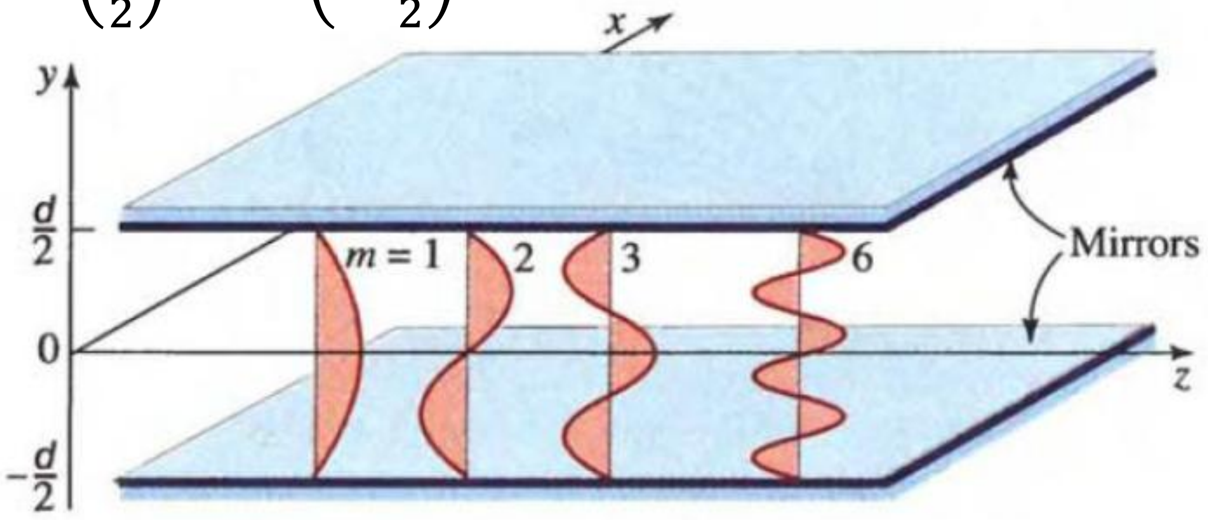
Waveguiding: Theory

- EM field in between two perfect mirrors
- Discrete number of solutions: **Helmholtz equations**

$$\nabla^2 \boldsymbol{E} + \boldsymbol{k_0^2} \varepsilon_r \boldsymbol{E} = \boldsymbol{0}$$

Assuming y direction: $\frac{\partial^2 \boldsymbol{E}}{\partial y^2} + \underbrace{\left(k_0^2 \varepsilon_r - \beta^2\right)}_{k_y^2} \boldsymbol{E} = \boldsymbol{0},$

$$E\left(\frac{d}{2}\right) = E\left(-\frac{d}{2}\right) = 0$$



Definition: EM fields which satisfy this equation, we call (eigen)modes!

$$\boldsymbol{E}(x, y, z) = \boldsymbol{E}_m(x, y) e^{j\beta z}$$

With

$$\boldsymbol{E_m} \begin{cases} A\cos\left(\frac{m\pi}{d} y\right) & m\ odd \\ A\sin\left(\frac{m\pi}{d} y\right) & m\ even \end{cases}$$

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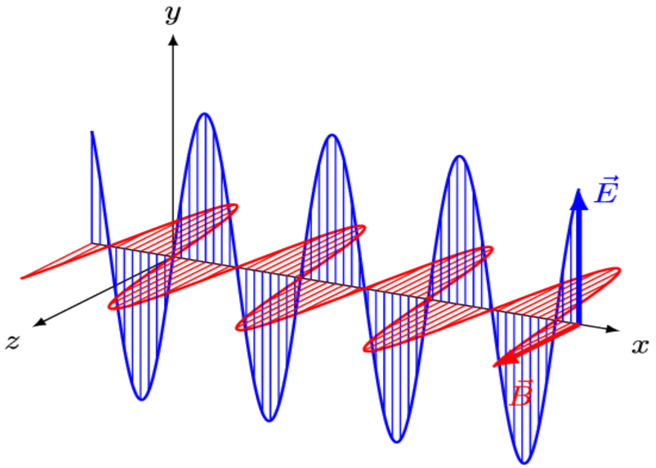
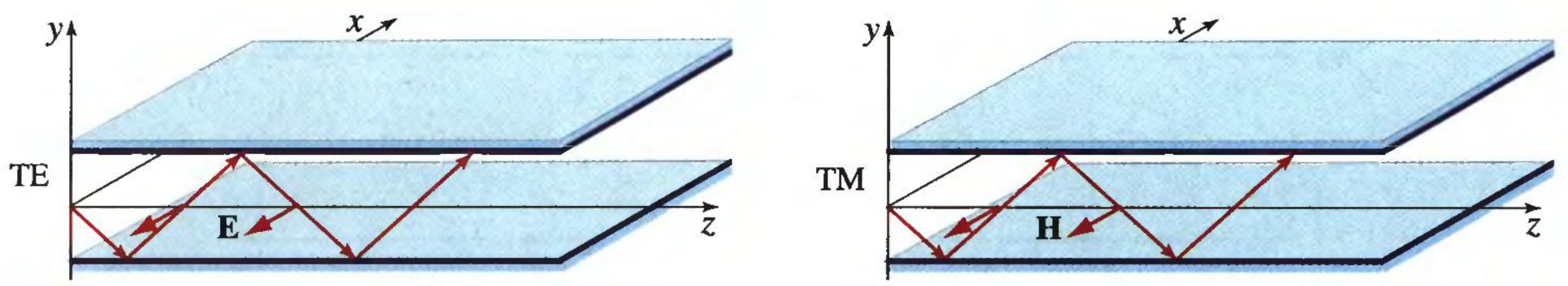
Waveguiding: Theory

- Planar dielectric waveguide
  - Core ($n_1, n_1 > n_2$)
  - Cladding (n_2)
- TE and TM modes





TE mode : *no electric field* in the propagation direction
TM mode: *no magnetic field* in the propagation direction

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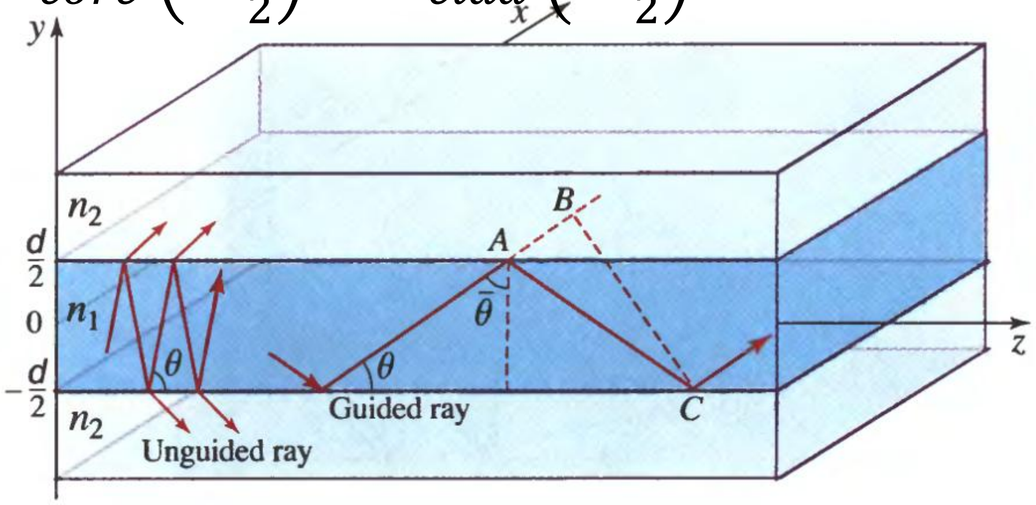
Waveguiding: Theory

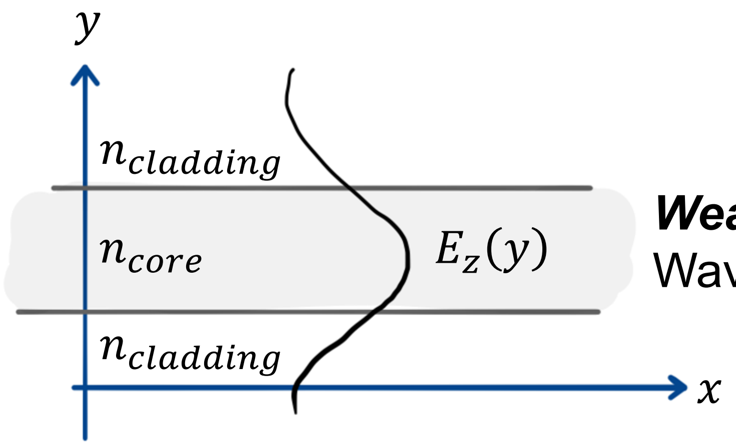
- Planar dielectric waveguide
  - Core ($n_1, n_1 > n_2$)
  - Cladding (n_2)

$$\frac{\partial^2 \boldsymbol{E}}{\partial y^2} + (k_0^2 \varepsilon_{core} - \beta^2)\, \boldsymbol{E} = \boldsymbol{0}$$

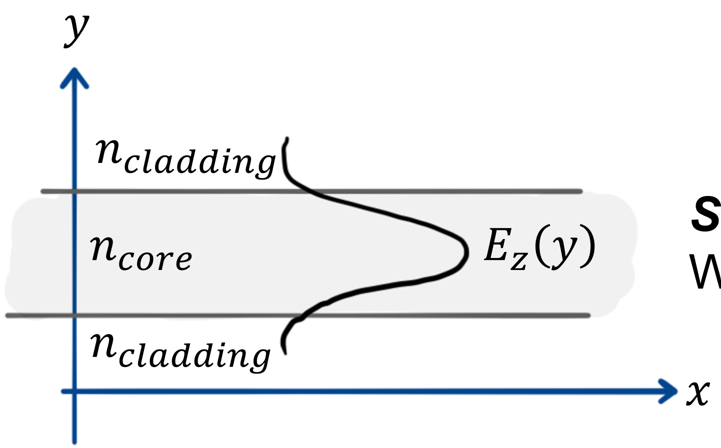
$$\frac{\partial^2 \boldsymbol{E}}{\partial y^2} + (k_0^2 \varepsilon_{cladding} - \beta^2)\, \boldsymbol{E} = \boldsymbol{0}$$

$$E_{core}\left(\pm\frac{d}{2}\right) = E_{clad}\left(\pm\frac{d}{2}\right)$$





Weakly confined
Wave leaks into cladding



Strongly confined
Wave mostly in the core

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P&S COMSOL® Design: Simulations of Optical Components Tutorial 4: Optical Waveguide I

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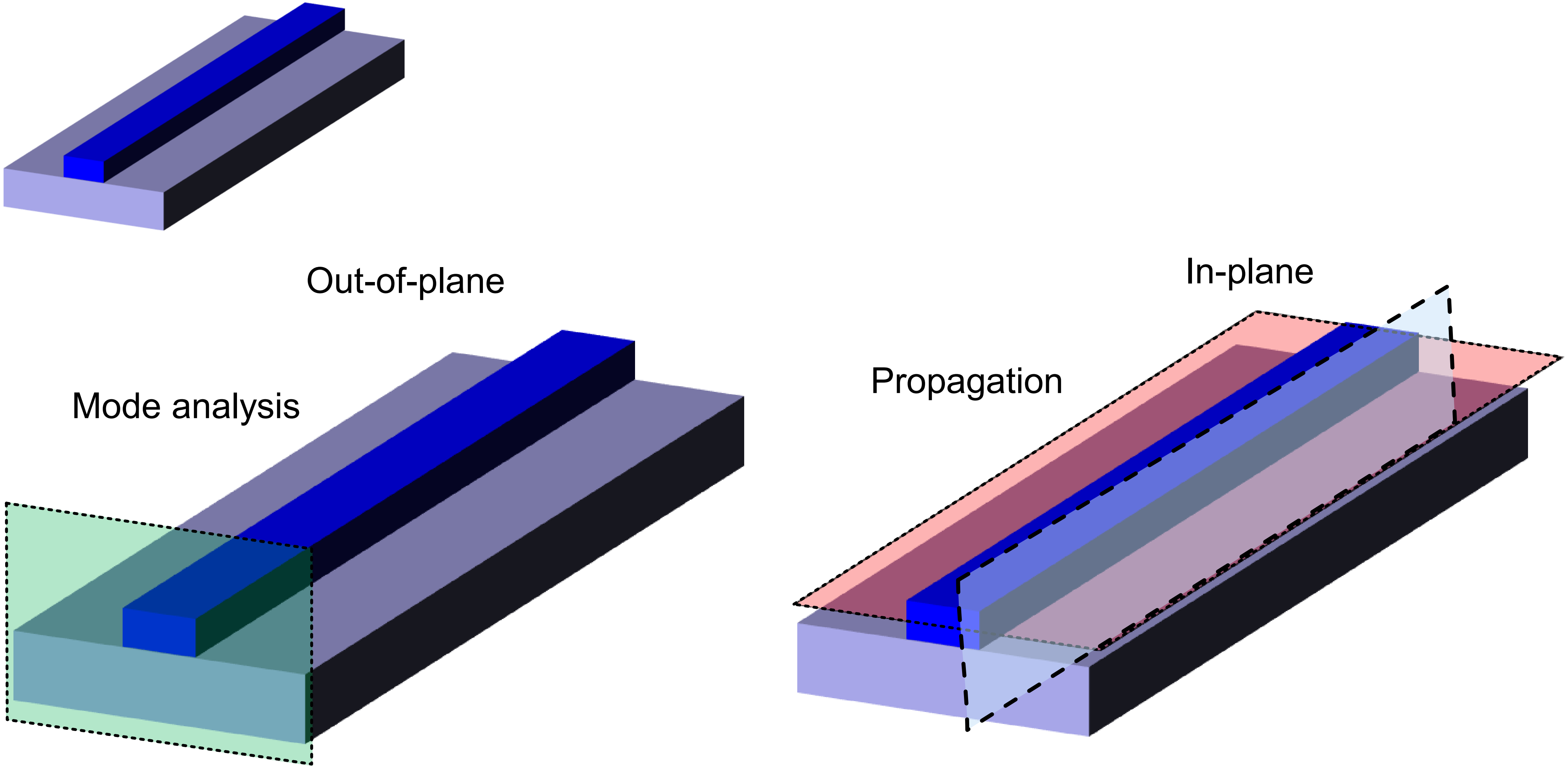
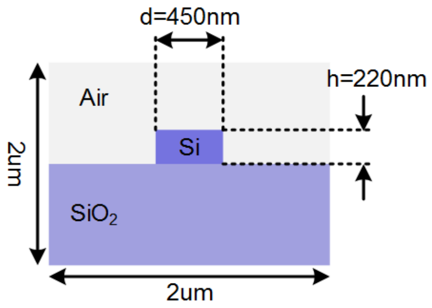
Waveguiding in COMSOL



Waveguiding in COMSOL

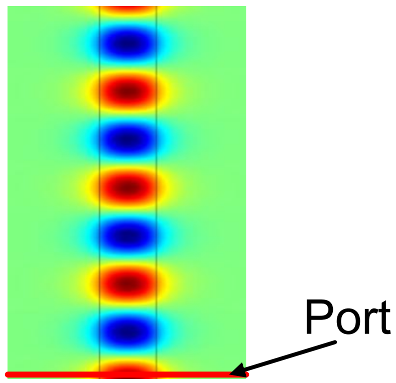
- Propagation
  - *Out-of-plane*: *Ports are not* defined (eigenvalue solution to the whole geometry)
  - *In-plane*: *Ports* need to be defined (eigenvalue solution to the defined port)

Next week



Out-of-plane ← *Excitation from surface plane* ← *Electromagnetics node: NO PORT*

TODAY



In-plane ← *Excitation from boundary* ← *Electromagnetics node: TE or TM*

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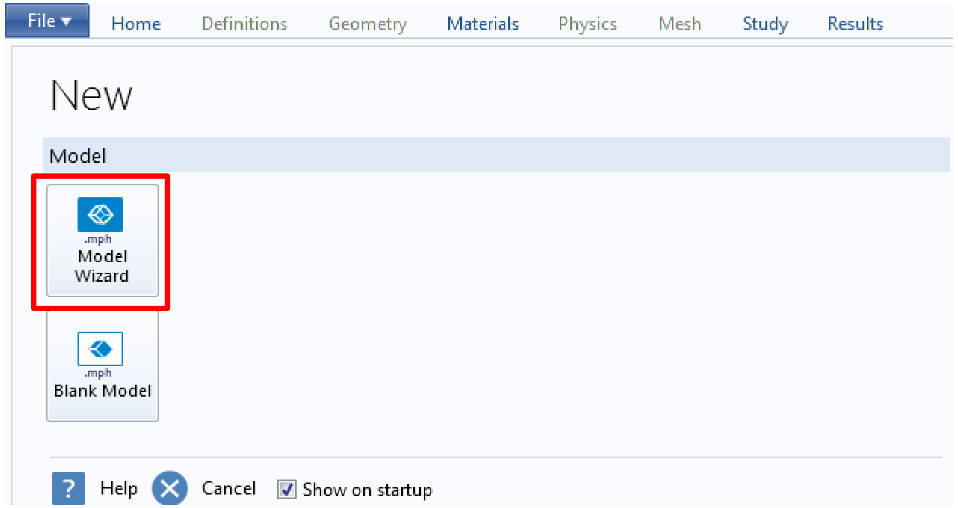
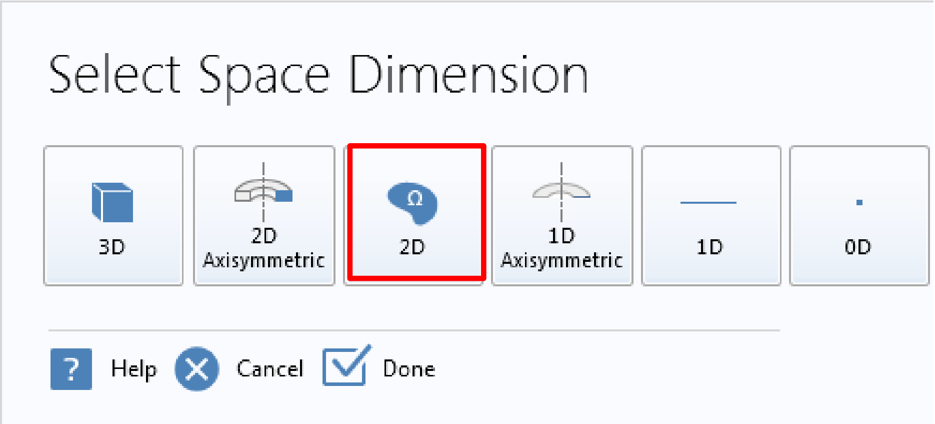
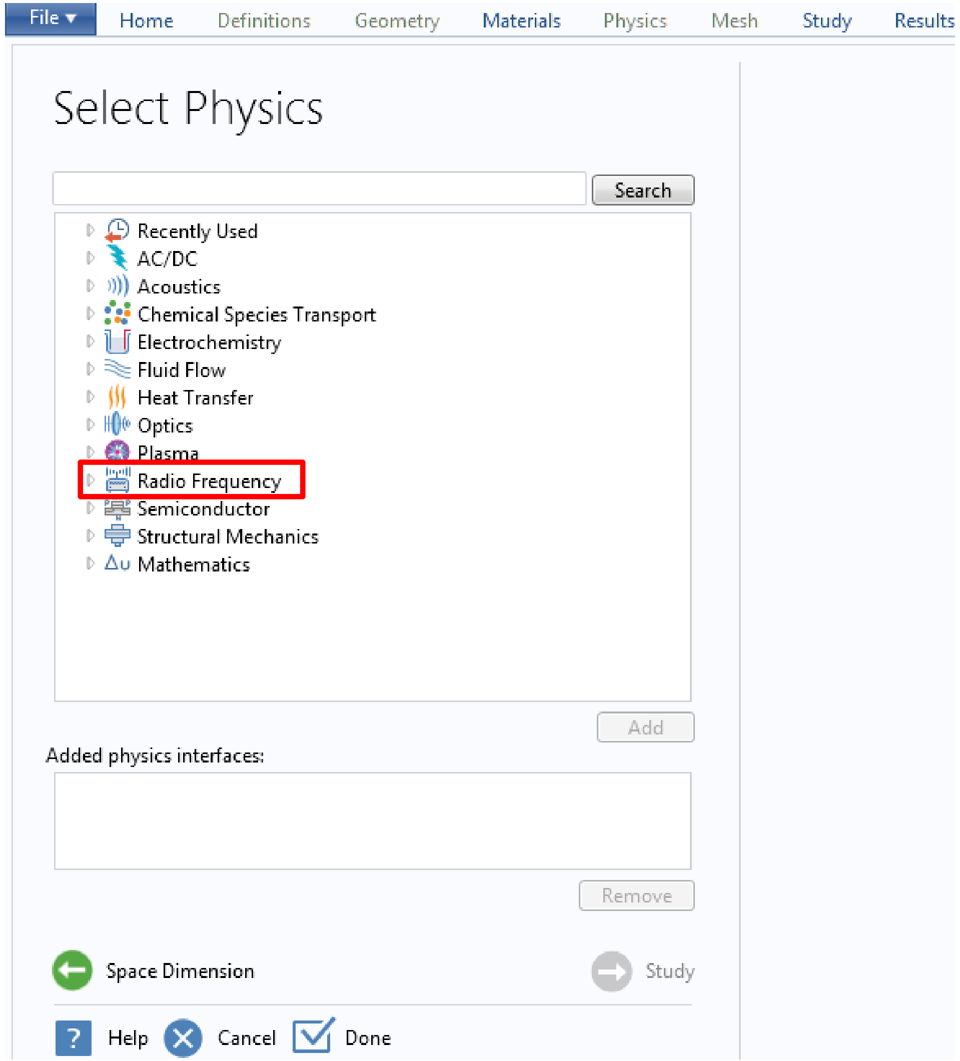
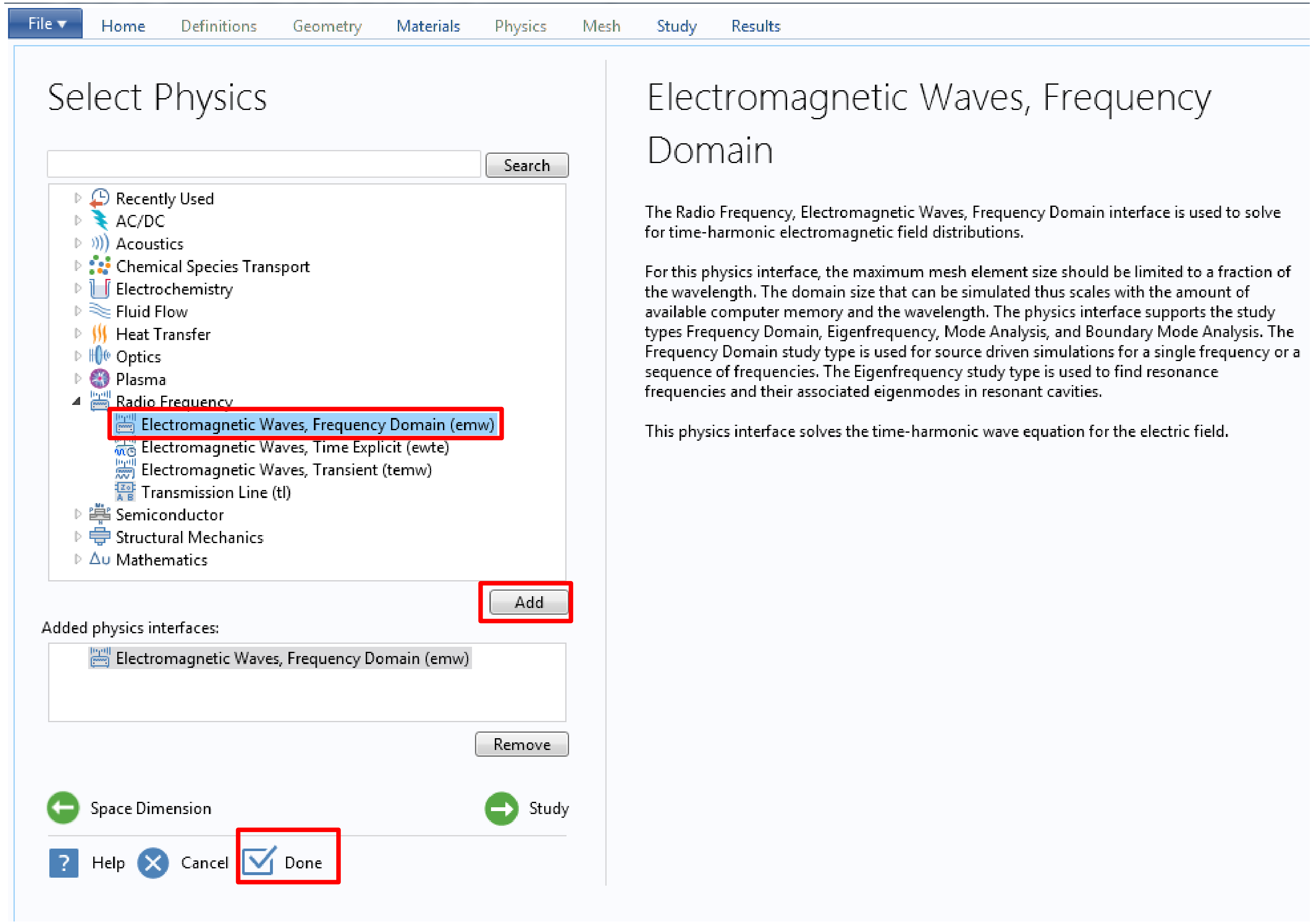
Set up Environment







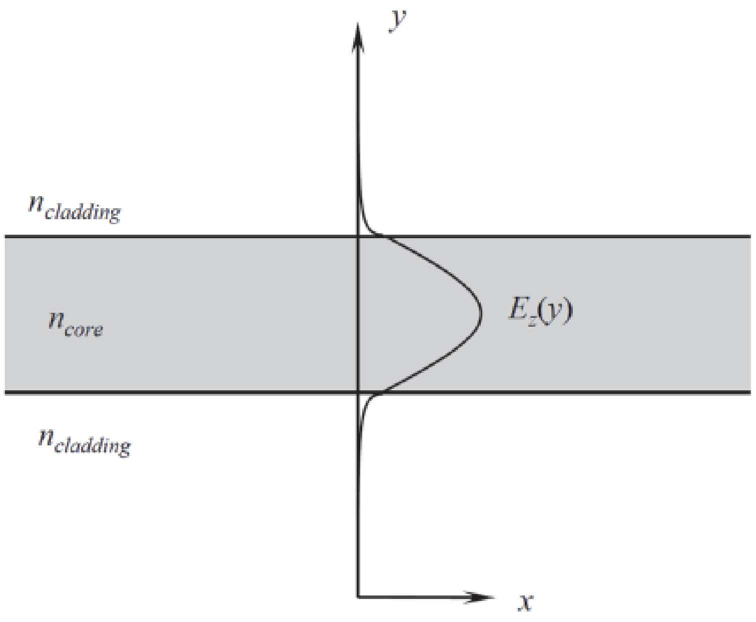
Set up Environment



Set up Environment

Parameters

Name	Expression	Value
lambda0	1550[nm]	1.55E-6 m
n_core	1.5	1.5
n_cladding	1	1
h_core	2.5[um]	2.5E-6 m
h_cladding	7[um]	7E-6 m
w_slab	50[um]	5E-5 m
f0	c_const/lambda0	1.9341449E14...



Define: Mesh & Port Definition & Study

- Port Definition
  - Left «excitation» port
  - Right «collection» port
  - Type of port: **Numeric**
- Mesh
  - Cladding: finer
  - Core: Custom → Maximum element size → lambda0/n_core/6

- Study
  - Study Steps → Boundary Mode Analysis (Has to be Step 1)
    - Set frequency
    - Port number: 1
    - Desired number of modes : 1
    - Search for modes around: **n_core**
  - Study Steps → Boundary Mode Analysis for Port 2 (port number changed to 2)

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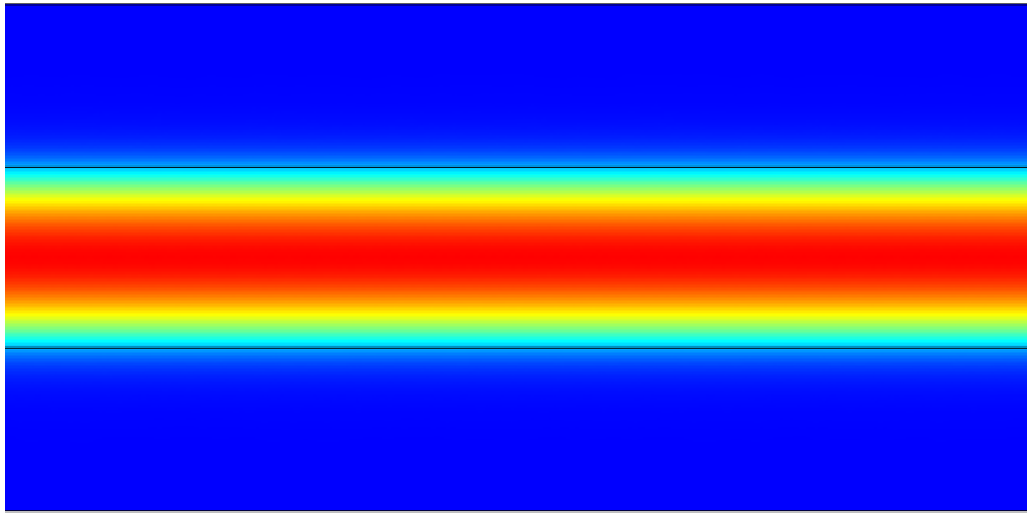
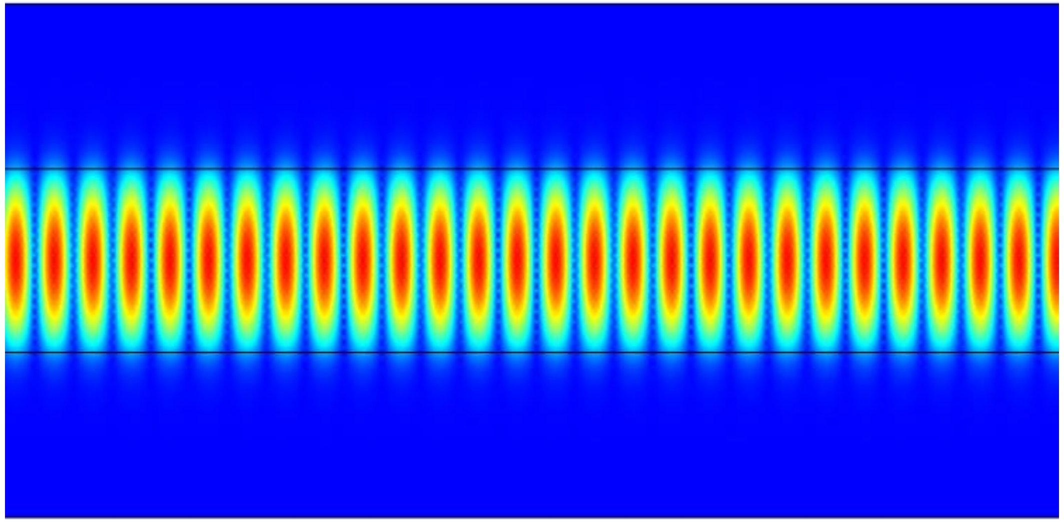
Single Mode Waveguide

Both images show the electric field's norm

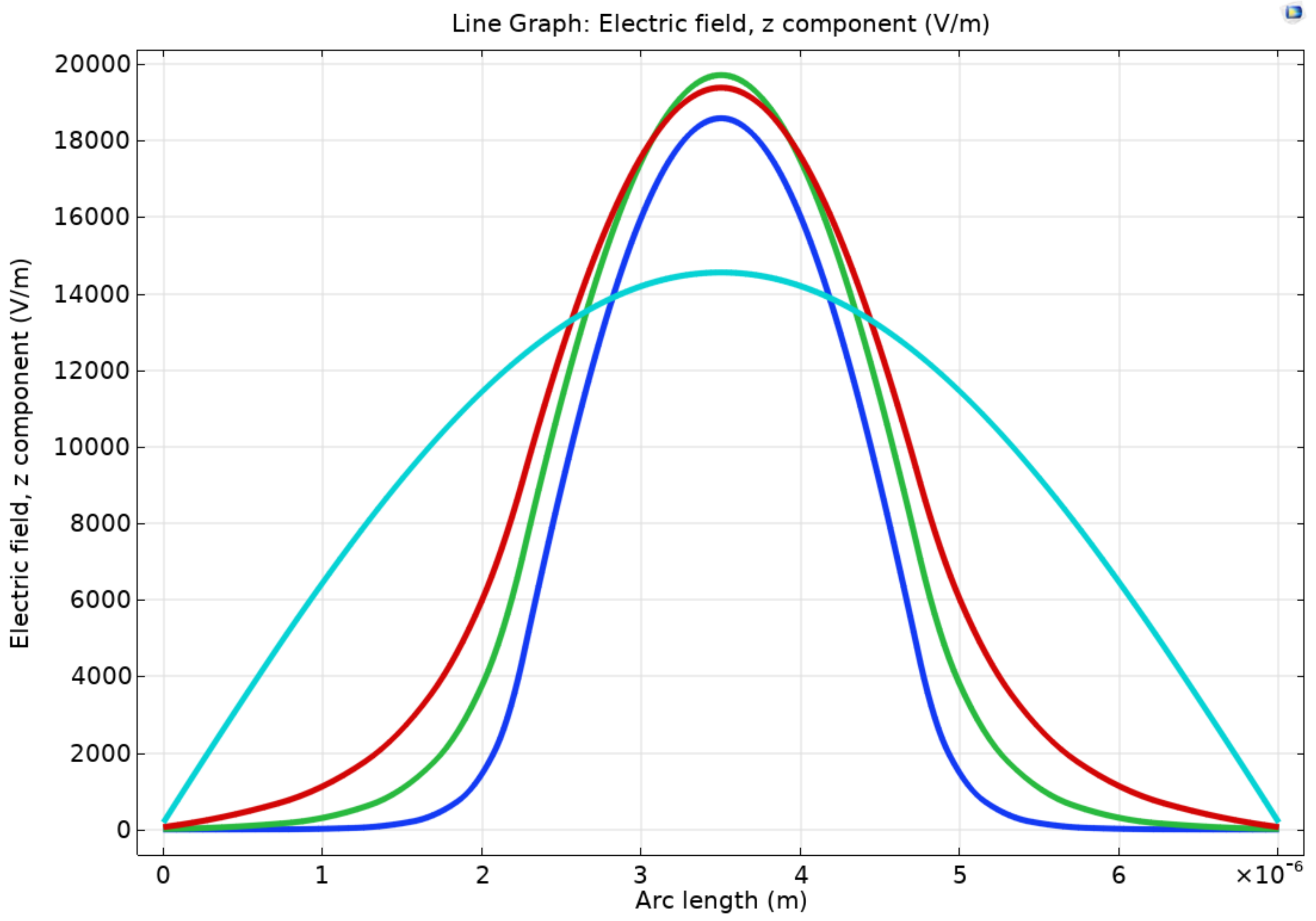
Which wave is propagating?
Which wave is standing?





Single Mode Waveguide

Which one would correspond to the highest core index? The lowest?



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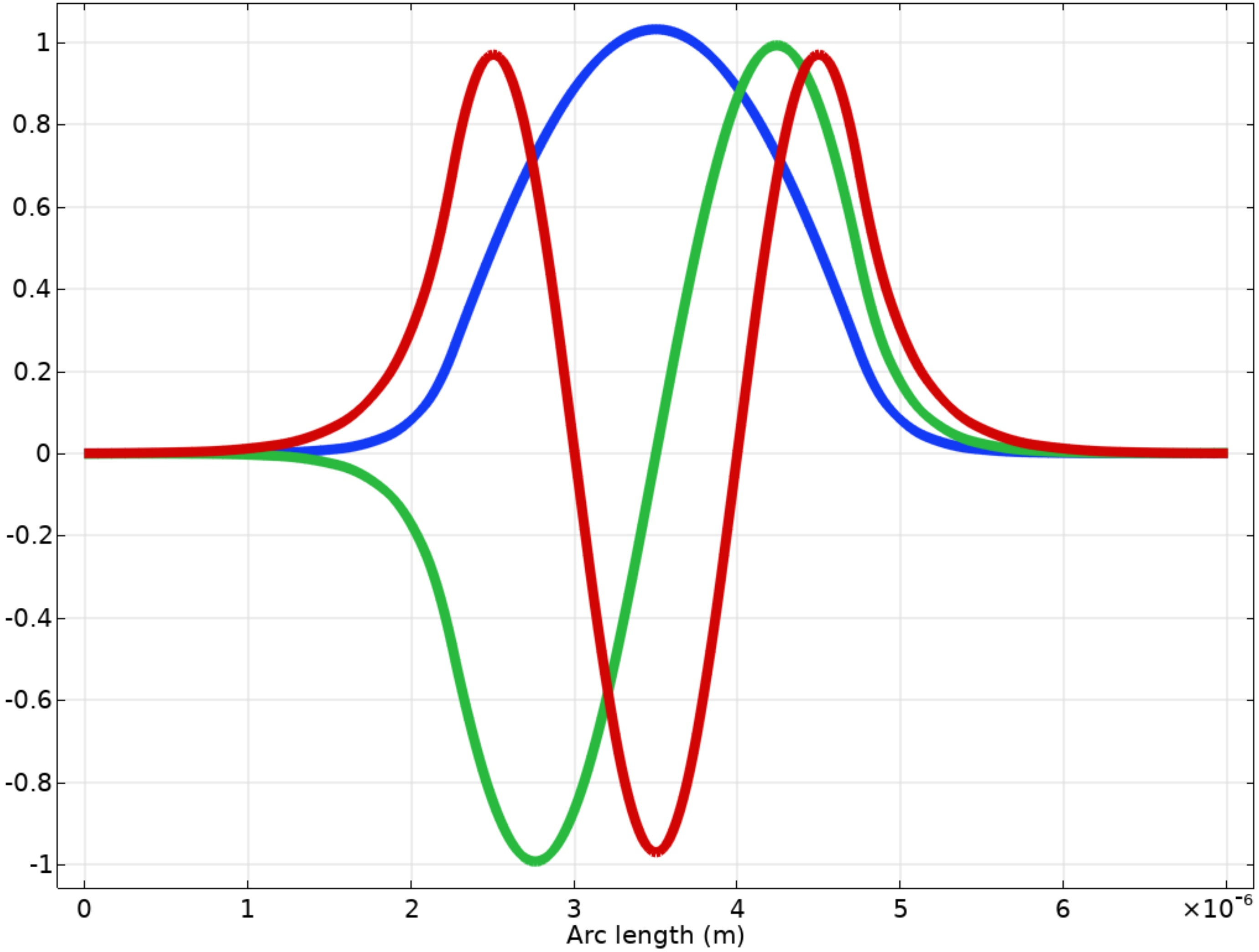
Multimode waveguide

Which one will have the highest neff? The Lowest? How can one increase/decrease the number of modes in a waveguide?



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