ETH zürich

P&S COMSOL® Design Tool

Week 2: EM Introduction & Introduction to COMSOL

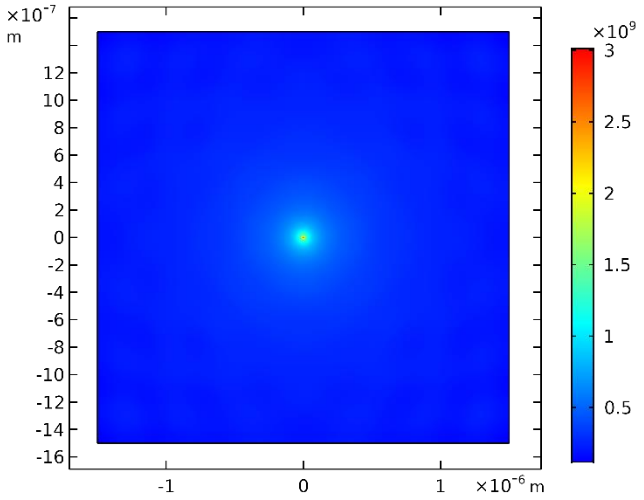
Taichiro Fukui, Maximilian Bosch

Recap

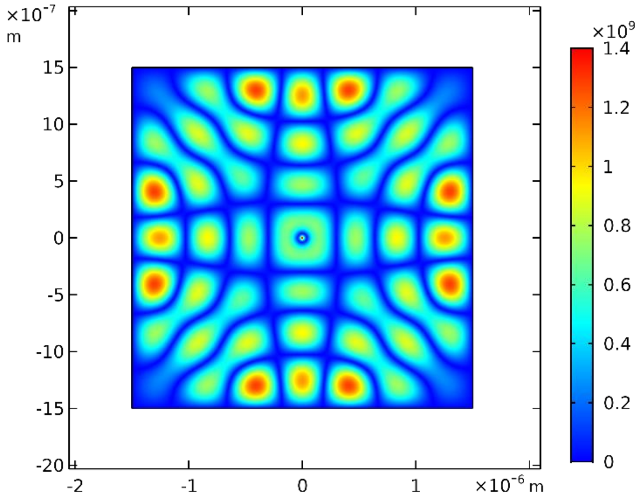
- Last Time
  - Introduction to COMSOL
  - Point Source model
    - Scattering Boundary Conditions
    - Perfect Electric Conduction (PEC)

- Today
  - More on Boundary Conditions
    - Perfect Electric Conductor (PEC)
    - Perfect Magnetic Conductor (PMC)
    - Scattering Boundary Condition
    - Periodic Boundary Condition (PBC)
    - Perfectly Matched Layer (PML)

Scattering

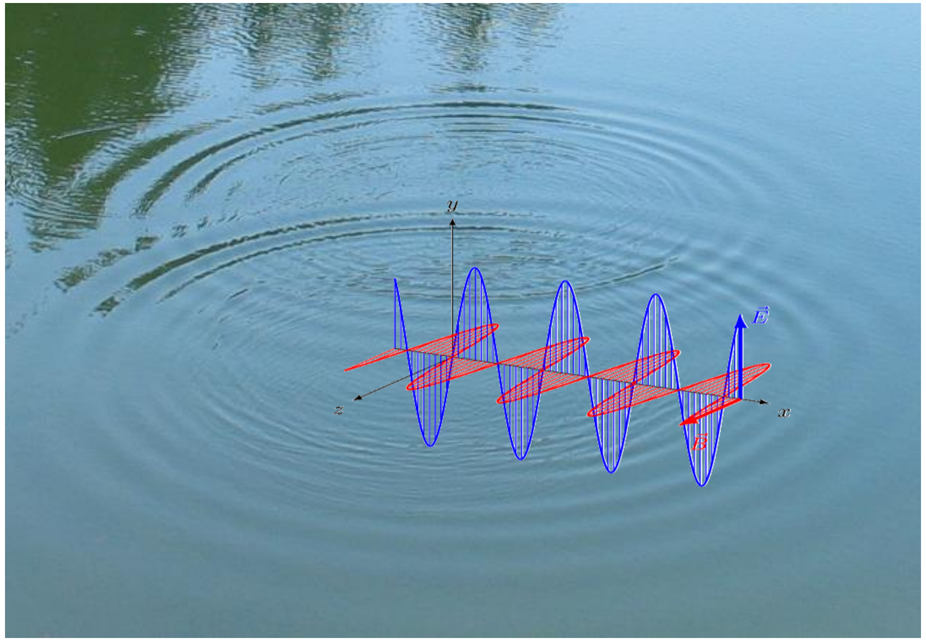


PEC



Motivation

- Wave behavior in nature
- But electromagnetic waves are (usually) invisible
- → COMSOL visualizes them!
- → Intuitive approach to understand mathematics
- Mathematics will be treated extensively in EM lectures



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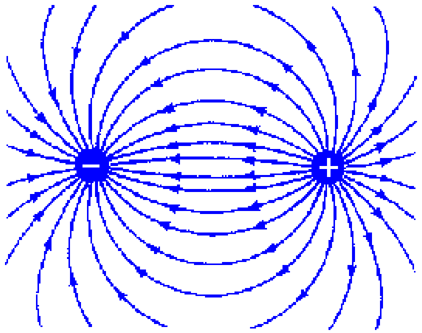
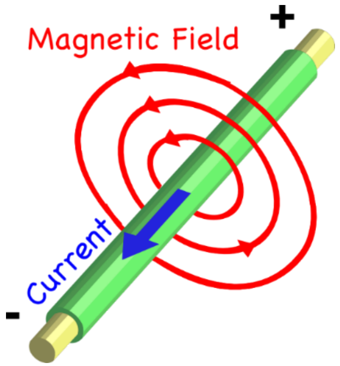
Electromagnetics: What you've seen before

- Physical phenomena and laws

Electric field between positive and negative charge

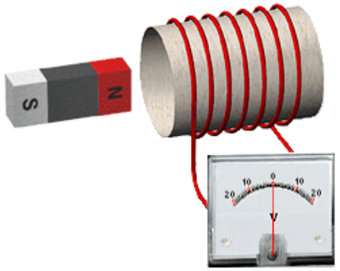


Current flows
→ magnetic field



Faraday's law of induction

$$u(t) = -\frac{d\Phi}{dt} = -\frac{d}{dt}\iint_A \vec{B}\cdot d\vec{A}$$



Coulomb force: $F = Eq$
Ohmic law: $U = R \cdot I$
Kirchhoff current & voltage law:

$$\sum_{node} I_i = 0, \sum_{loop} U_i = 0$$

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Electromagnetics: Maxwell's Equations (local form)

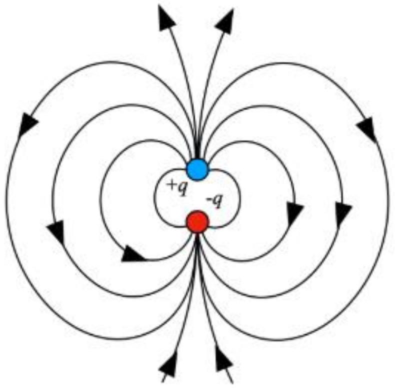
- Physical phenomena and laws

Gauss's law:

$$\nabla \cdot D = \rho$$

Electric displacement Field D

(Free) charge density ρ

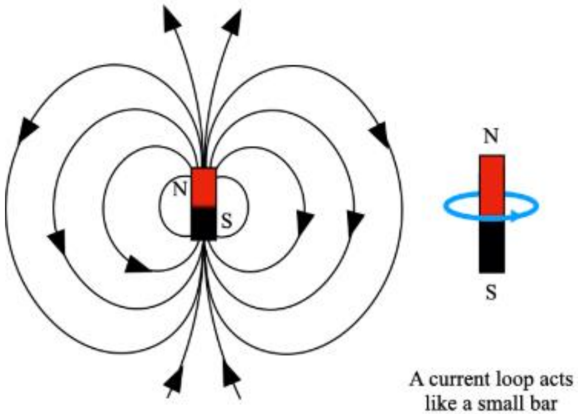


Electric dipole and electric field

Gauss's law (Magnetism):

$$\nabla \cdot B = 0$$

Magnetic Flux Density B



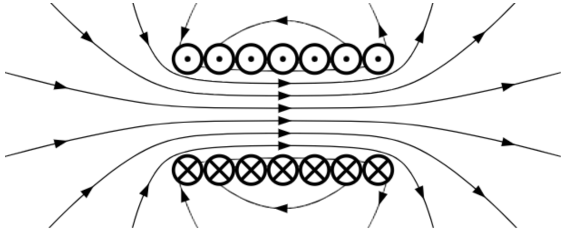
Magnetic dipole and magnetic field

A current loop acts like a small bar magnet

Faraday's law:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Electric Field E



Ampère's law:

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

(Free) current density J

Magnetic Field H

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Electromagnetics: Maxwell’s Equations (global form)

- Physical phenomena and laws

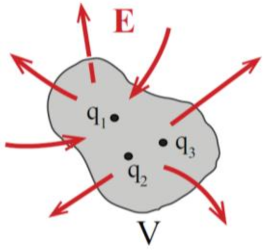
By using $\int_V \nabla \cdot \boldsymbol{F}\, dV = \oint_{\partial V} \boldsymbol{F} \cdot \boldsymbol{dA}$ *and* $\int_A (\nabla \times \boldsymbol{F}) \cdot \boldsymbol{dA} = \oint_{\partial A} \boldsymbol{F} \cdot \boldsymbol{dl}$

Gauss’s law:

$$\oint_{\partial V} D\, dA = \int_V \rho dV$$

Electric displacement Field D

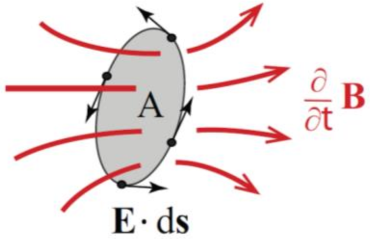
(Free) charge density ρ



Faraday’s law:

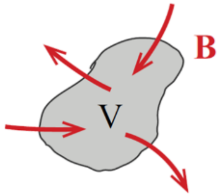
$$\oint_{\partial A} E\, ds = -\frac{\partial}{\partial t} \int_A B\, dA$$

Electric Field H



Gauss’s law (Magnetism):

$$\nabla \cdot B = 0 \Leftrightarrow \oint_{\partial V} B\, dA = 0$$



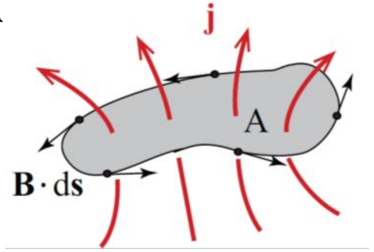
Magnetic Flux Density B

Ampère’s law:

$$\oint_{\partial A} H\, dl = \int_A \left(J + \frac{\partial D}{\partial t} \right) dA$$

(Free) current density J

Magnetic Field H



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Electromagnetics: Maxwell's Equations

- How do these equation describe light propagation in materials?
- One needs a model that captures the impact of fields on the material

- Electric Case   Polarization P

$$D = \varepsilon_0 E + P(E) \approx \varepsilon_0 E + \varepsilon_0 \chi_E^{(1)} E$$

- Magnetic Case   Magnetization M

$$B = \mu_0 H + M(H) \approx \mu_0 H + \mu_0 \chi_H^{(1)} H$$

$$\varepsilon_r = 1 + \chi_E^{(1)}$$
$$\mu_r = 1 + \chi_M^{(1)}$$

In linear case

Nonlinear Optics: 227-0655-00L

Electromagnetics: Materials Relations

- To analyze EM problems, we need to define the **material properties** involved
- Materials are defined by their **refractive index $\boldsymbol{n}$** which is defined as
  - $\boldsymbol{n} = \sqrt{\boldsymbol{\mu_r \varepsilon_r}}$, for vacuum $n = 1$
  - n is a complex number $n = n' + jk$

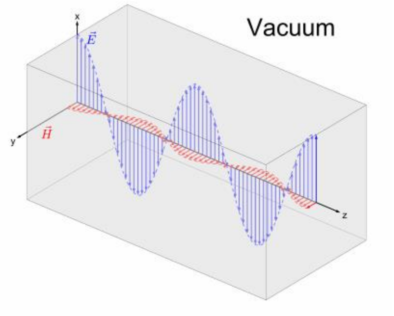
n': Influences wavelength

k: Influences losses

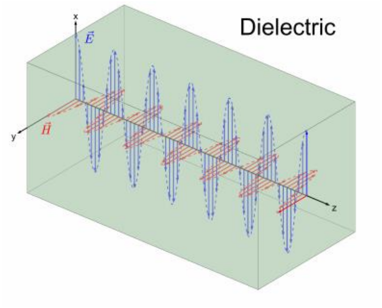
$$\varepsilon_r = 1 + \chi_E^{(1)}$$

$$\mu_r = 1 + \chi_M^{(1)}$$

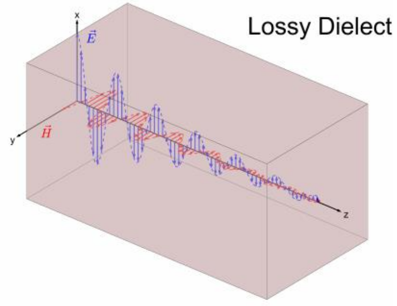
$n = 1$



$n = 3$



$n = 3 + 2j$



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Electromagnetics: Maxwell's Equations

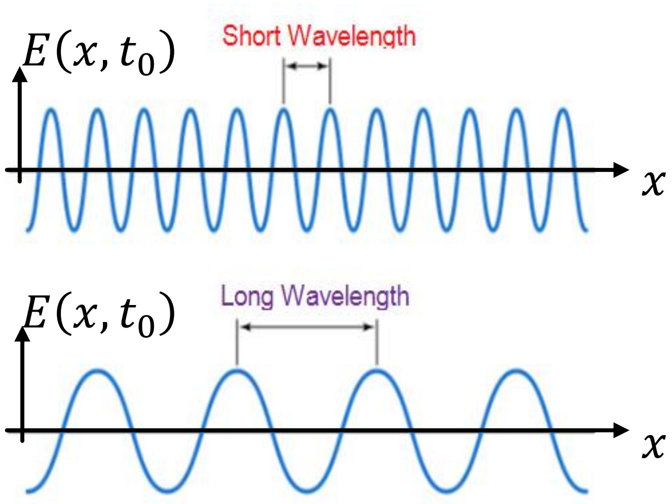
- Wave Equation from Maxwell's Equations
  - Homogenous, Isotropic, linear Material,
  - No Sources

$$\frac{\partial^2 E_z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} = 0$$

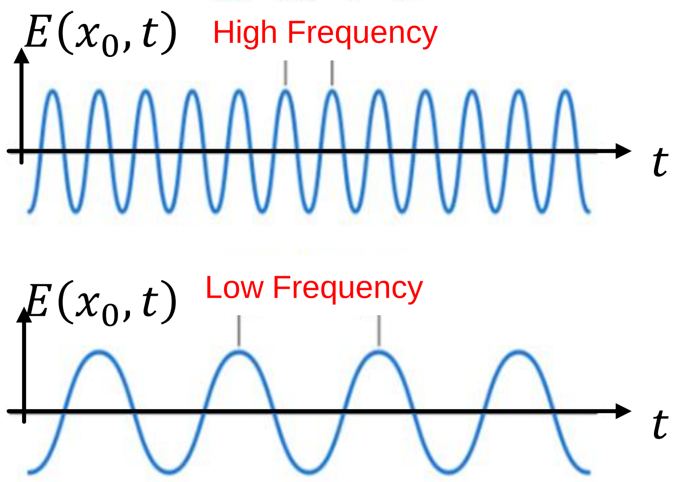
- General Solution

$$E_z(x,t) = g(ct \pm x)$$

<u>Fixed time</u>



<u>Fixed Location</u>



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Electromagnetics: Wavelength and Frequency

$$\frac{\partial^2 E_z}{dx^2} - \frac{1}{c^2}\frac{\partial^2 E_z}{dt^2} = 0$$

- Plane wave Solution:

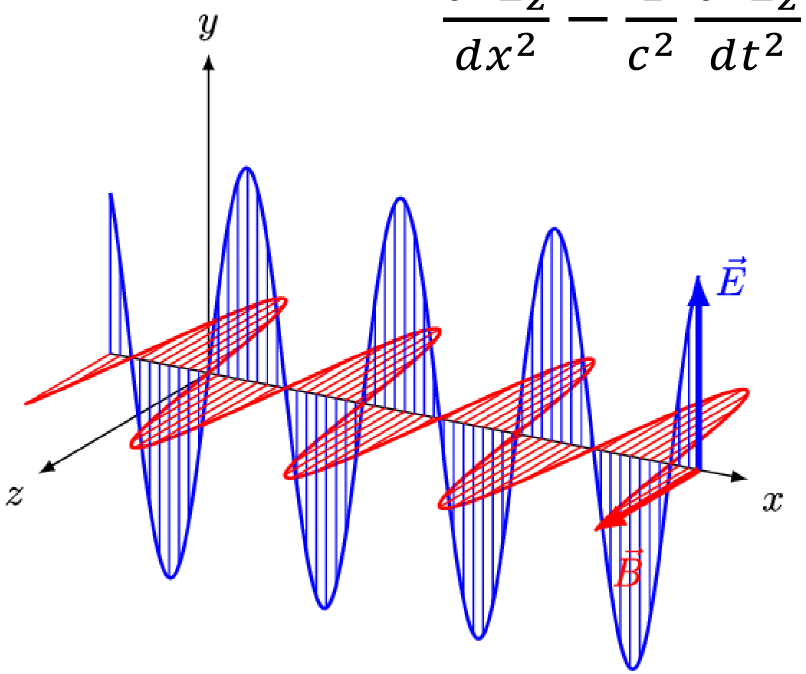
$$E_z(x,t) = E_0 e^{j\omega t} e^{-jkx}$$

- Proof:

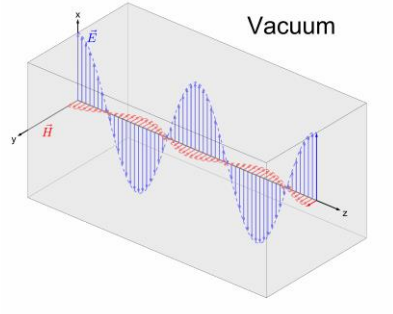
$$-k^2 e^{j\omega t} e^{-jkx} + \frac{\omega^2}{c^2} e^{j\omega t} e^{-jkx} = 0$$

- With

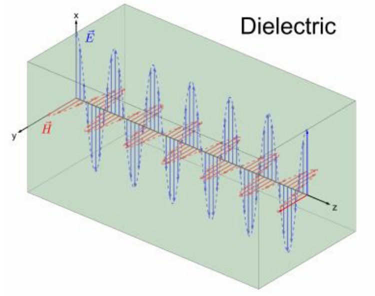
$$k^2 = \frac{\omega^2}{c^2} = \frac{\omega^2}{c_0^2} n^2$$



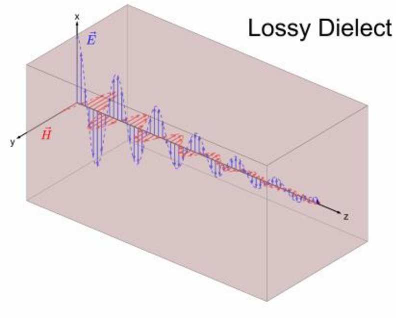
$n = 1$



$n = 3$



$n = 3 + 2j$



And what does Comsol solve?

The wave equation: $\nabla \times \mu_r^{-1}(\nabla \times \boldsymbol{E}) - k_0^2\left(\varepsilon_r - \frac{j\sigma}{\omega\varepsilon_0}\right)\boldsymbol{E} = \boldsymbol{0}$

First step: Insert magnetization into the Faraday's equation $\nabla \times \boldsymbol{E} = -\mu_0\mu_r\frac{\partial \boldsymbol{H}}{\partial t} \Leftrightarrow \mu_r^{-1}(\nabla \times \boldsymbol{E}) = -\mu_0\frac{\partial \boldsymbol{H}}{\partial t}$

Second step: Apply the curl's operation $\nabla \times \left(\mu_r^{-1}(\nabla \times \boldsymbol{E})\right) = \nabla \times \left(-\mu_0\frac{\partial \boldsymbol{H}}{\partial t}\right) = -\mu_0\nabla \times \left(\frac{\partial \boldsymbol{H}}{\partial t}\right) = -\mu_0\frac{\partial}{\partial t}(\nabla \times \boldsymbol{H})$

Third step: Insert local Ohm's law and polarization into Ampere's equation

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} = \sigma\boldsymbol{E} + \varepsilon_0\varepsilon_r\frac{\partial \boldsymbol{E}}{\partial t} \Leftrightarrow \nabla \times \left(\mu_r^{-1}(\nabla \times \boldsymbol{E})\right) = -\mu_0\frac{\partial}{\partial t}\left(\sigma\boldsymbol{E} + \varepsilon_0\varepsilon_r\frac{\partial \boldsymbol{E}}{\partial t}\right) = -\mu_0\sigma\frac{\partial \boldsymbol{E}}{\partial t} - \mu_0\varepsilon_0\varepsilon_r\frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

Fourth step: Solve the equation into the frequency domain: $\frac{\partial}{\partial t} \leftrightarrow j\omega$

$$\nabla \times \left(\mu_r^{-1}(\nabla \times \boldsymbol{E})\right) = \boldsymbol{E}(-j\mu_0\omega\sigma + \mu_0\varepsilon_0\varepsilon_r\omega^2) \Leftrightarrow \nabla \times \mu_r^{-1}(\nabla \times \boldsymbol{E}) - k_0^2\left(\varepsilon_r - \frac{j\sigma}{\omega\varepsilon_0}\right)\boldsymbol{E} = \boldsymbol{0}$$

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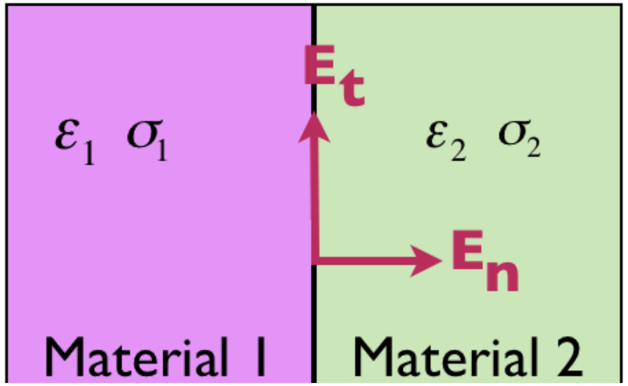
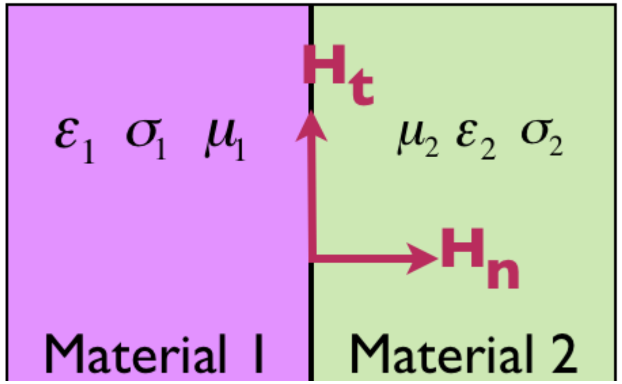
Boundary Conditions

- Maxwell's Boundary Conditions





Boundary Conditions

$E_{t,1} - E_{t,2} = 0$

$\varepsilon_1 E_{n,1} - \varepsilon_2 E_{n,2} = \rho_s$ (surface charge density at the interface)

$H_{t,1} - H_{t,2} = J_s$ (surface current density at the interface)

$\mu_1 H_{n,1} - \mu_2 H_{n,2} = 0$

227-0160-00L: Fundamentals of Physical Modeling and Simulations

227-0110-00L: Electromagnetic Waves: Materials, Effects, and Antennas:

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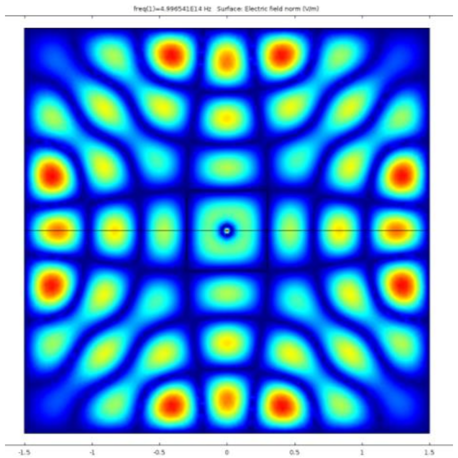
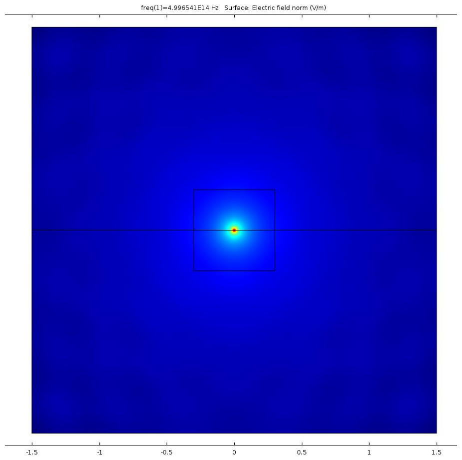
COMSOL: Boundary Conditions

- Purpose of boundary conditions → define simulation domain

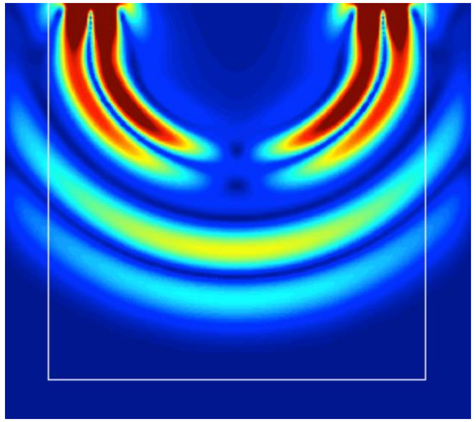
- Types of boundary conditions in COMSOL
  - Perfect Electric Conductor (PEC)
  - Perfect Magnetic Conductor (PMC)
  - Scattering Boundary Condition (SBC)
  - Periodic Boundary Condition (PBC)
  - Perfectly Matched Layer (PML)



PEC Boundaries



Scattering Boundaries



PML Boundaries

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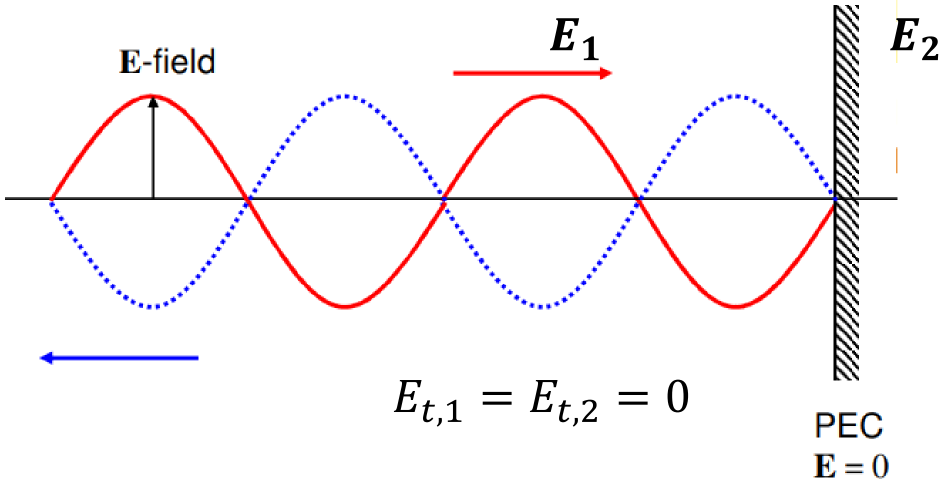
COMSOL: Boundary Conditions

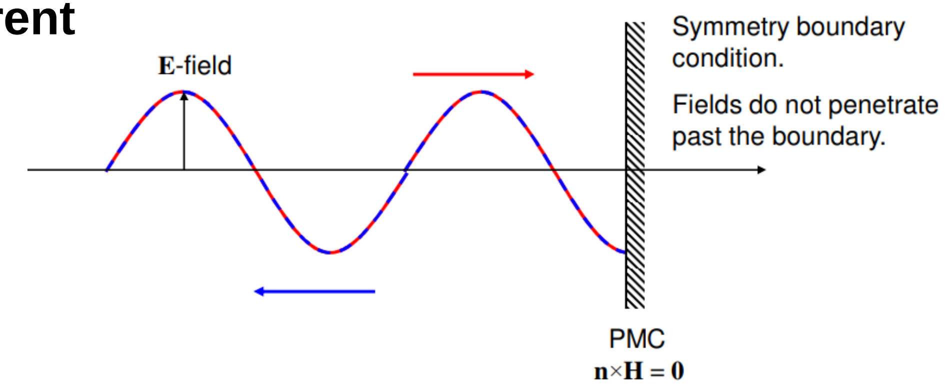
- **Perfect Electric Conductor (PEC)**
  - Properties
    - Equivalent to infinite electric conductivity
    - $\boldsymbol{J} = \sigma \boldsymbol{E_2} \Leftrightarrow \boldsymbol{E_2} = \frac{\boldsymbol{J}}{\sigma} \xrightarrow[\sigma \to \infty]{} \boldsymbol{0}$: no electric outside the domain, **the E field is back reflected to the domain**

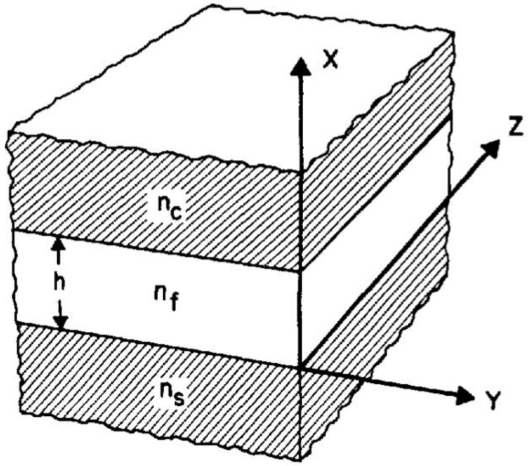


  - Examples
    - Microwave planar structures
    - Metallic substrates
    - Short circuit interface
    - To implement some certain symmetry (antisymmetrical **E** field, symmetrical **H** field)

COMSOL: Boundary Conditions

- **Perfect Magnetic Conductor (PMC)**
  - Properties
    - Equivalent to infinite magnetic conductivity (large μ)
    - $\boldsymbol{B} = \mu \boldsymbol{H_2} \Leftrightarrow \boldsymbol{H_2} = \frac{\boldsymbol{B}}{\mu} \xrightarrow[\mu \to \infty]{} \boldsymbol{0}$: no electric outside the domain, **the H field is back reflected to the domain, No surface current**



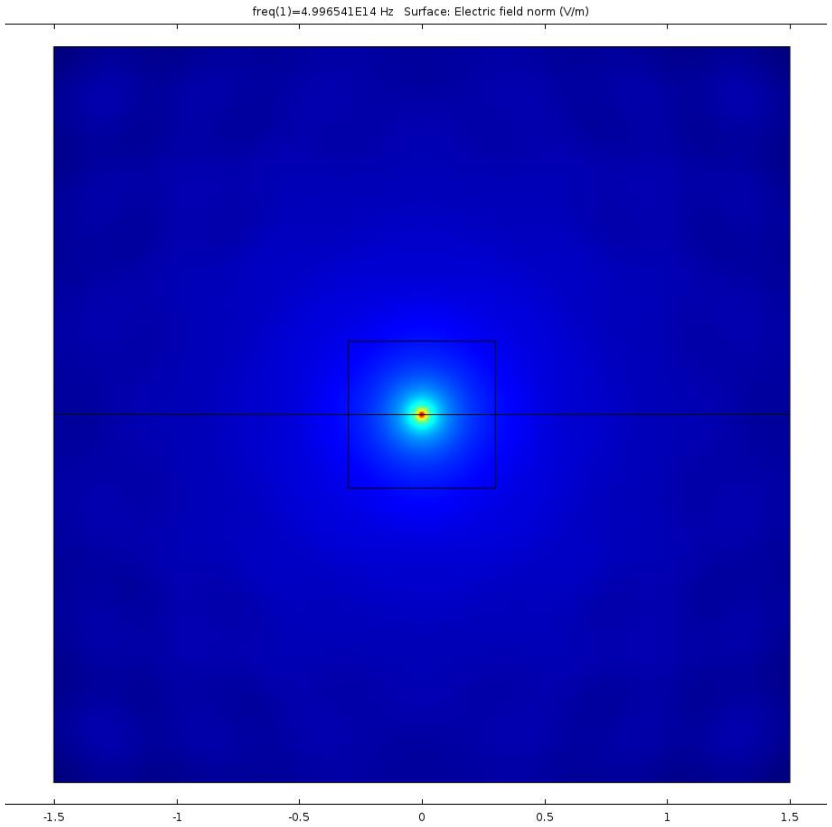


  - Examples
    - Interface between dielectric and air
    - Open circuit interface (high impedance)
    - To account for certain symmetry (symmetrical **E** field, antisymmetrical **H** field)

$$H_{t,1} = H_{t,2} = 0$$

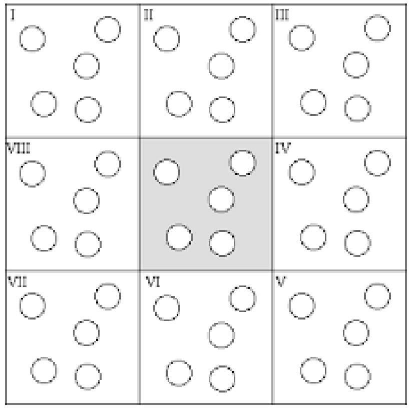
COMSOL: Boundary Conditions

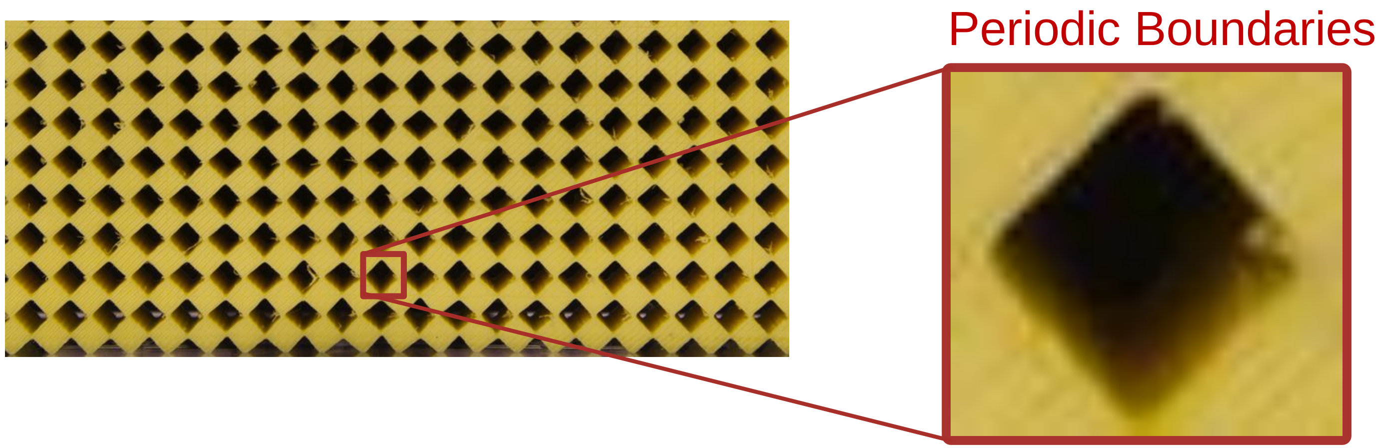
- **Scattering Boundary Condition**
  - Properties
    - Electric field is absorbed → no reflection
  - Examples
    - To simulate an “infinite” domain
    - As we did last time with the point source

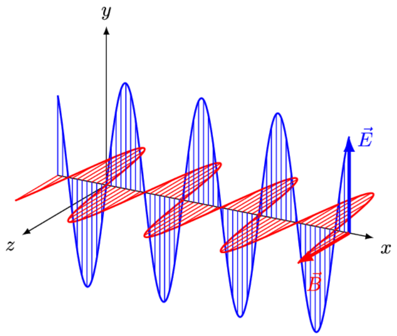


COMSOL: Boundary Conditions

- **Periodic Boundary Condition**
  - For repeating structures
  - Use a unit cell for the analysis
  - Simulates systems expanding infinitely in 1D/2D



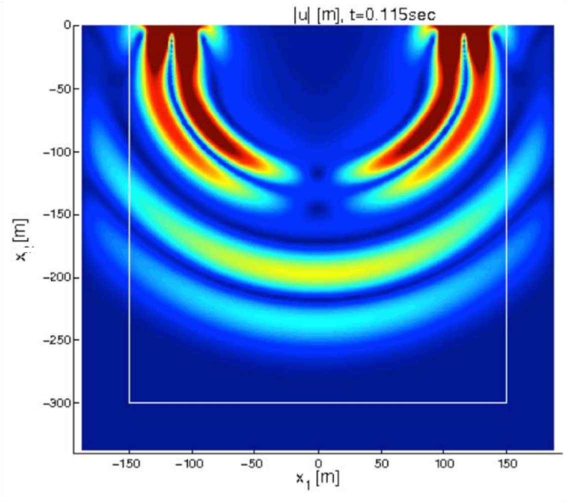


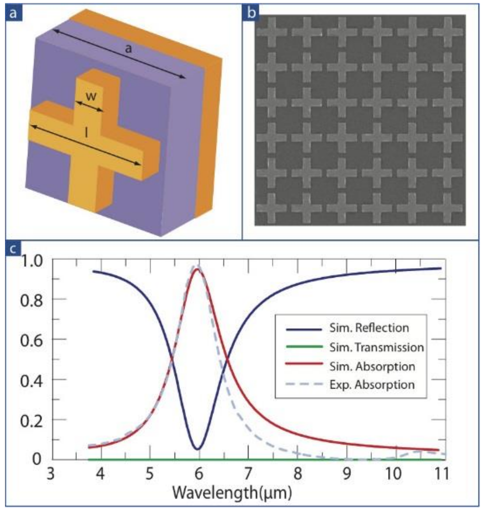


COMSOL: Boundary Conditions

- **Perfectly Matched Layer (PML)**
  - Scattering Boundary
  - Properties
    - Absorbing boundary
    - Truncates EM space in numerical simulations.
    - Possible to simulate open boundary problems.
    - Reduce scattered/reflected waves.

  - Examples
    - Antennas
    - Reflectors
    - Absorbing structures
    - Metamaterials





Real life absorber

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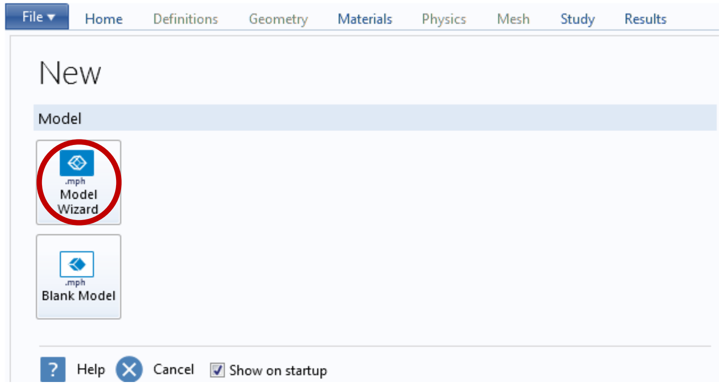
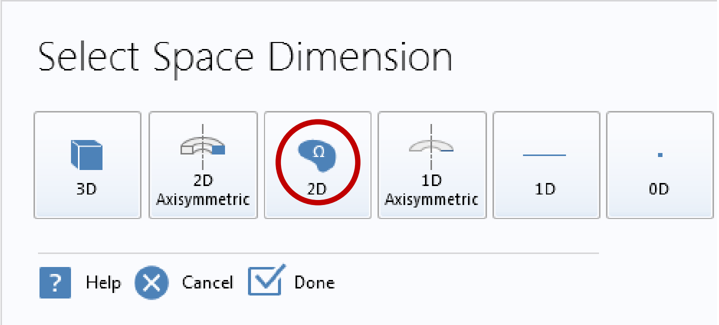
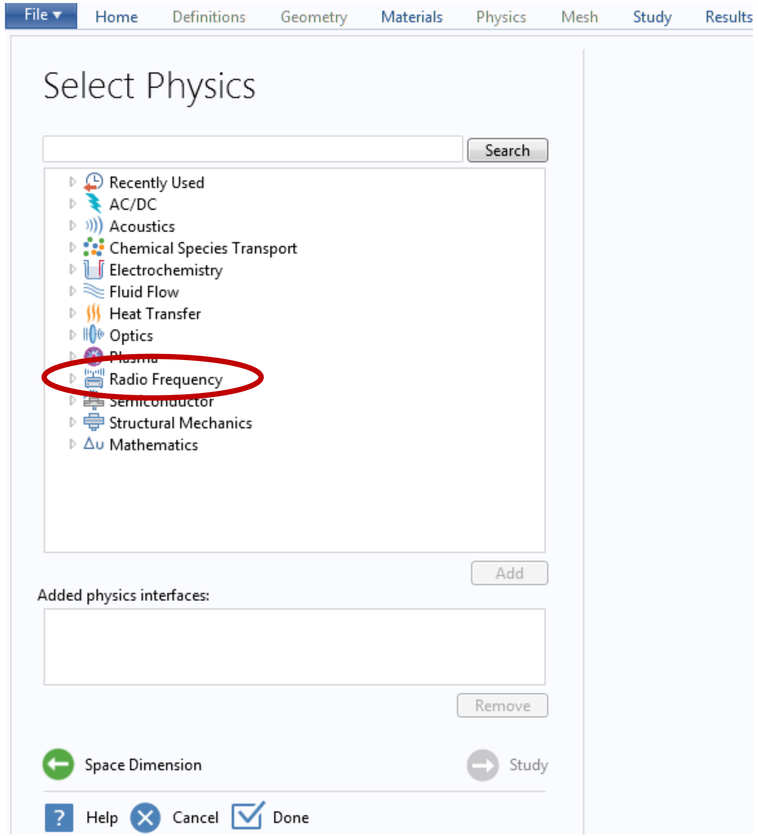
COMSOL: Last Time

Command to start Comsol: comsol







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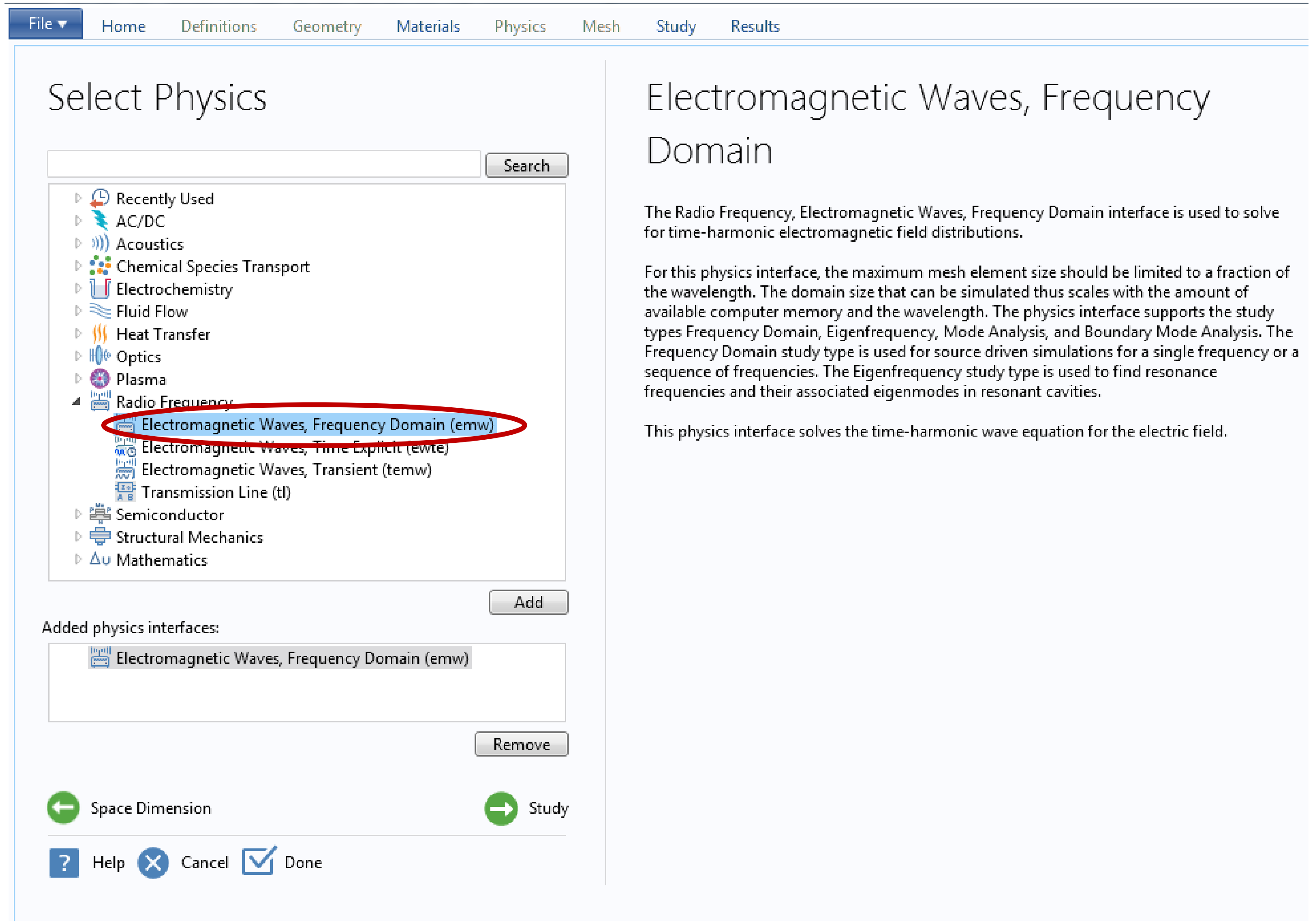
COMSOL: Last Time



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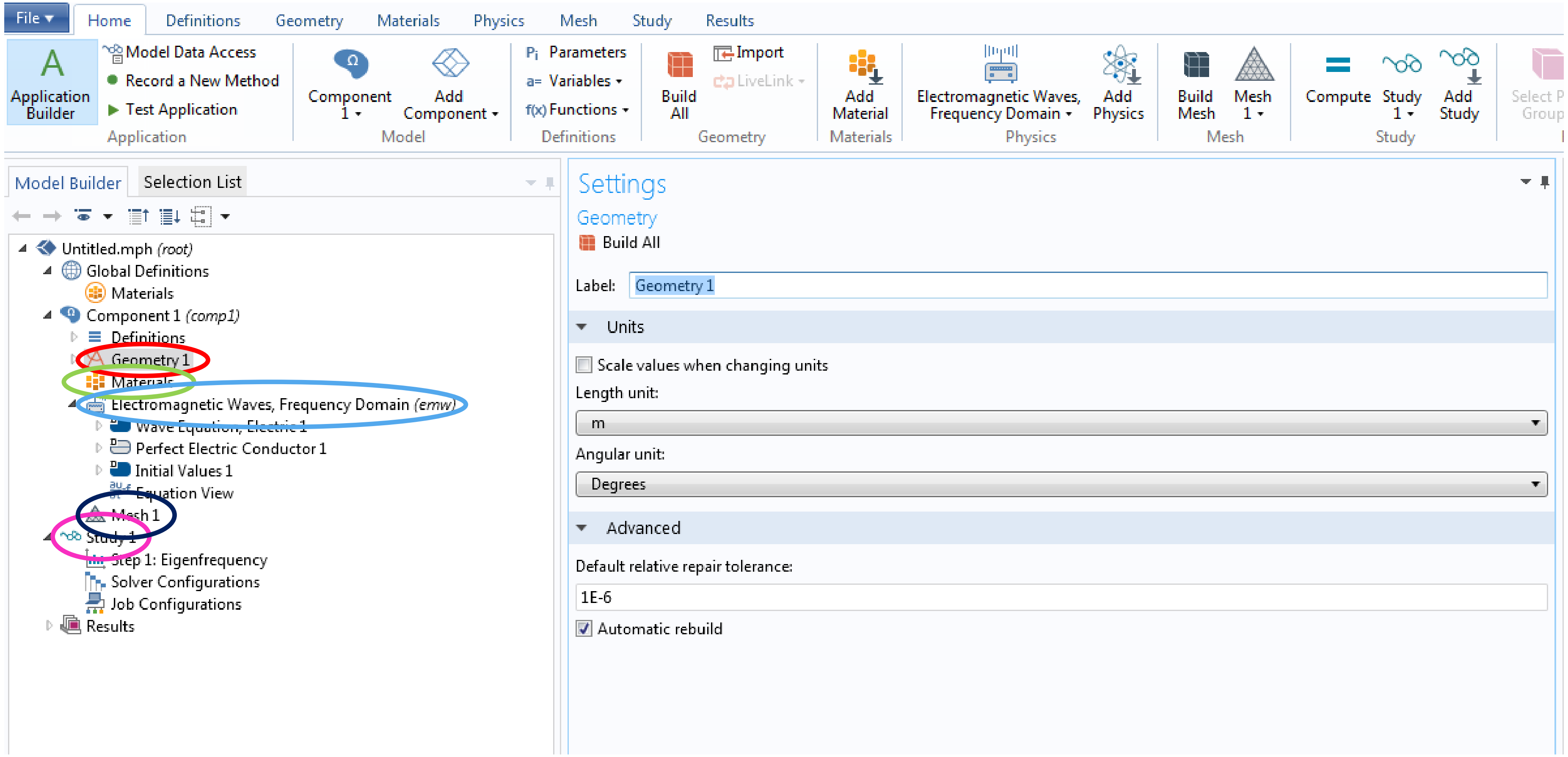
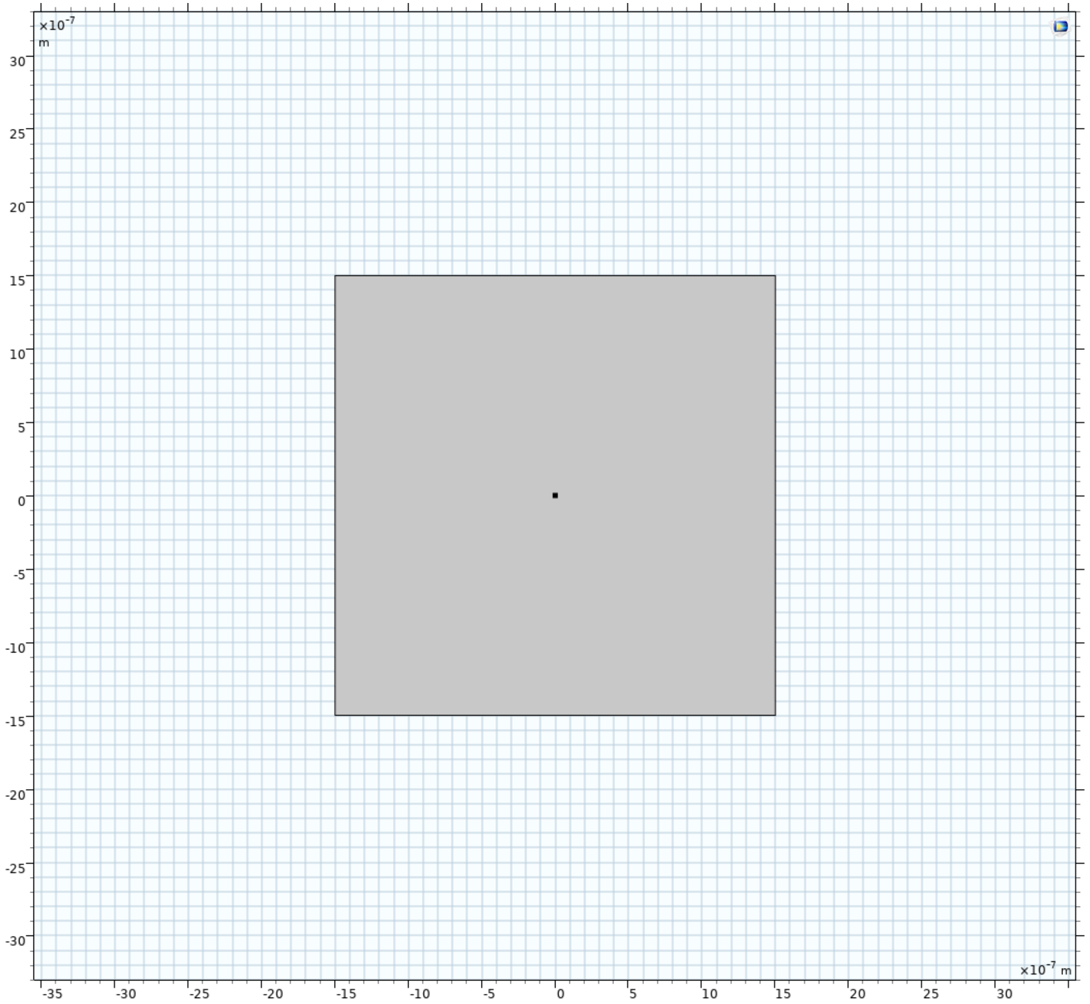


COMSOL: Last Time

- Define simulation domain
- Build geometry



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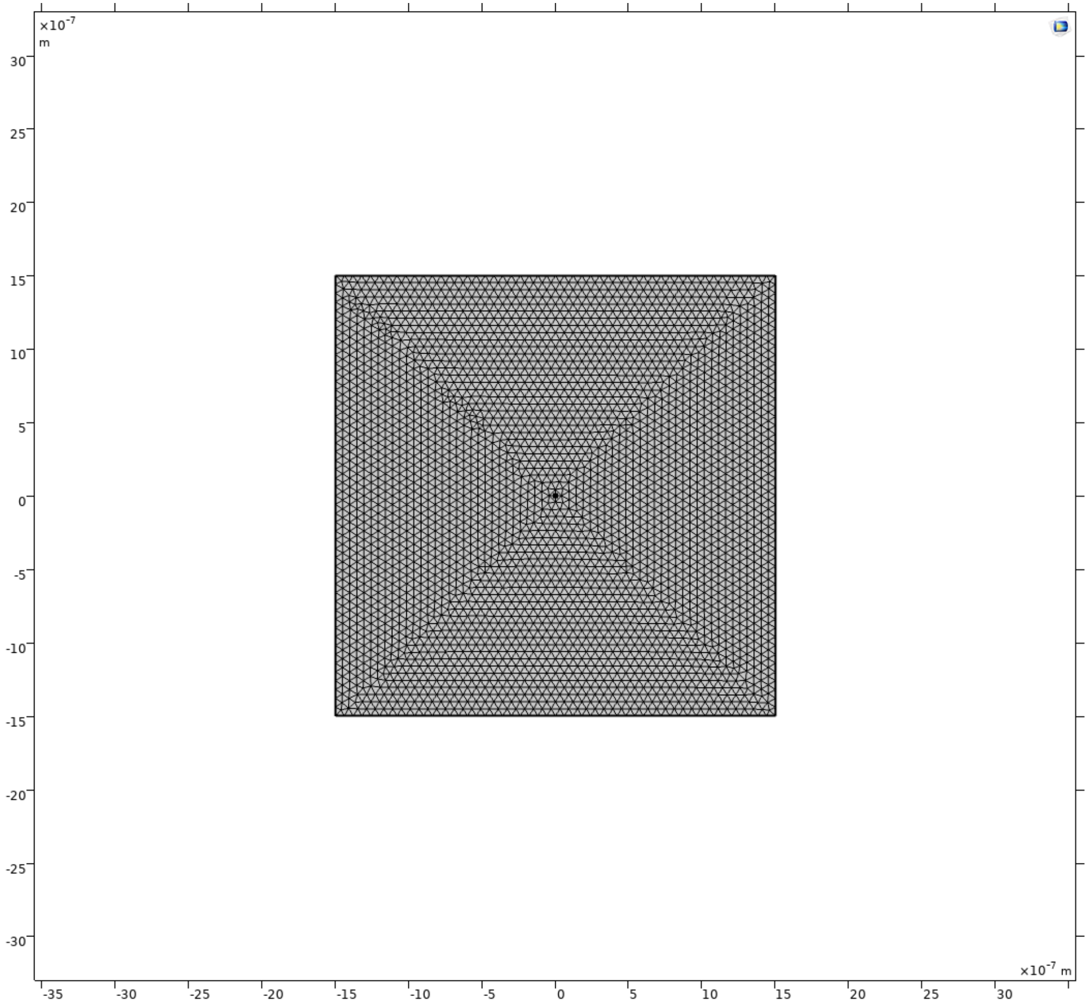
COMSOL: Last Time

- Discretization of simulation domain
  - Build mesh
- Mesh size determines accuracy of solution
  - Too large mesh ⇒ wrong results
- Accuracy vs. simulation time
  - **Today**: Optimization by manual refinement



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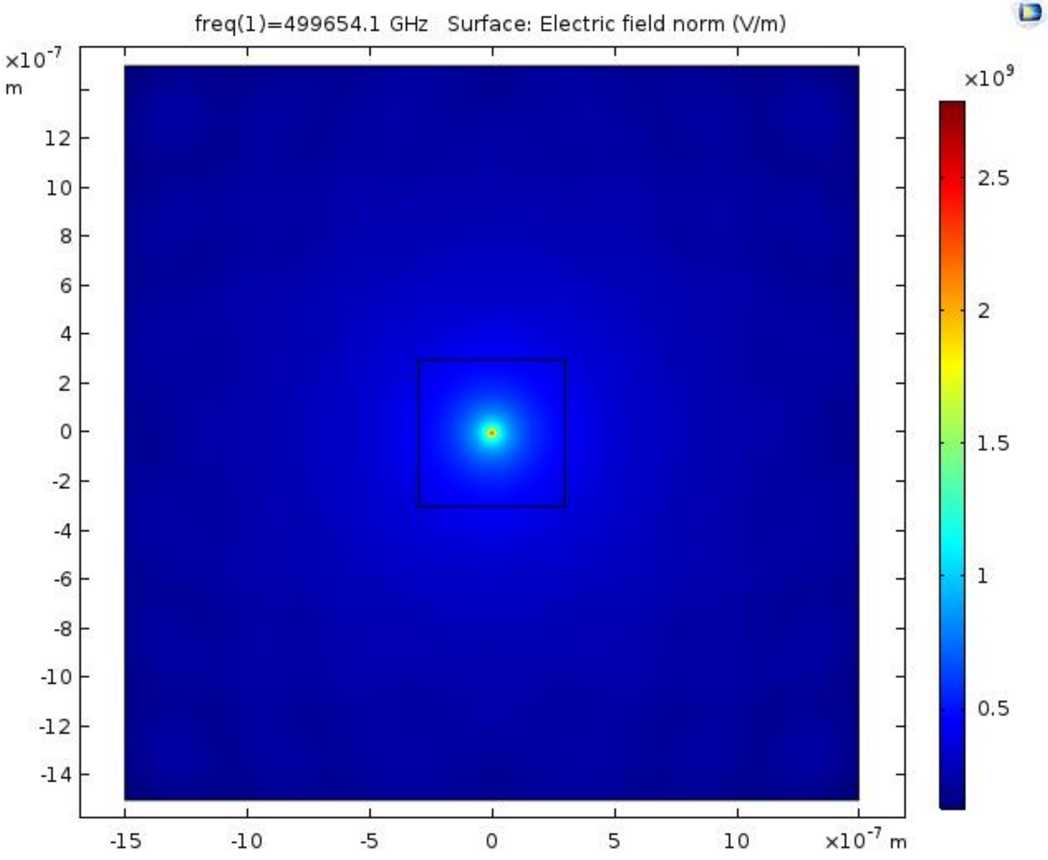
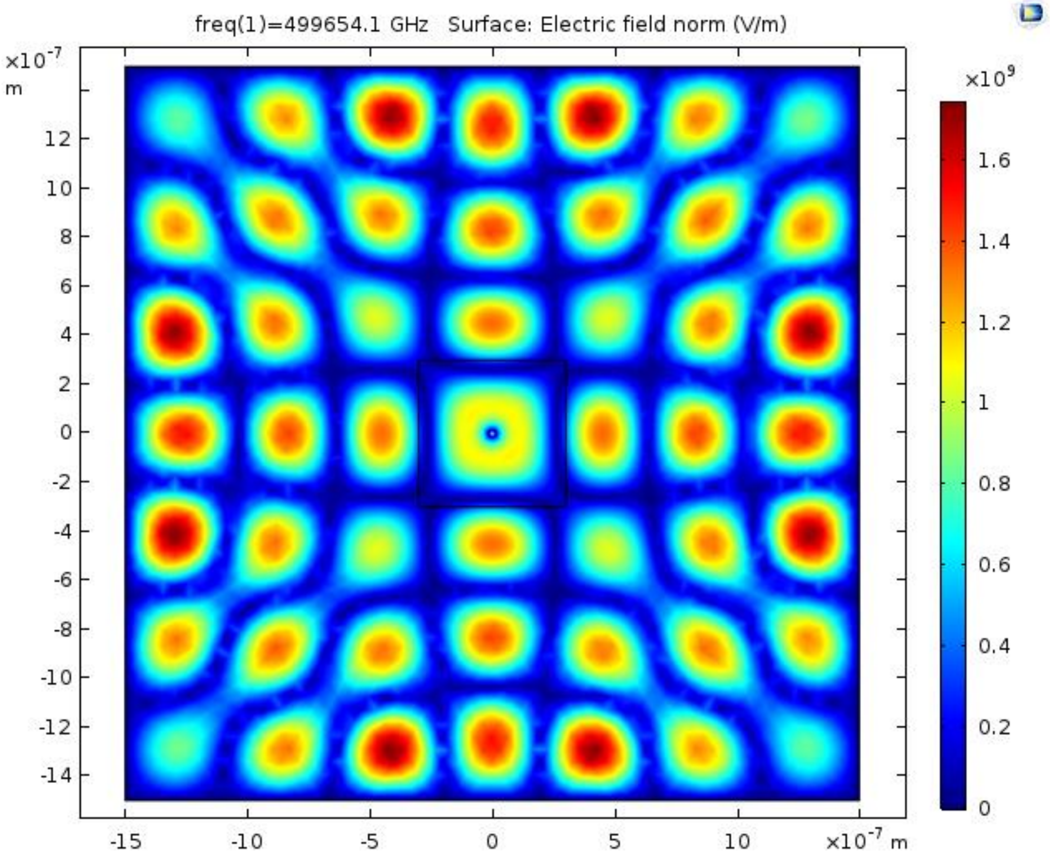
COMSOL: Last Time



Scattering Boundaries



PEC Boundaries

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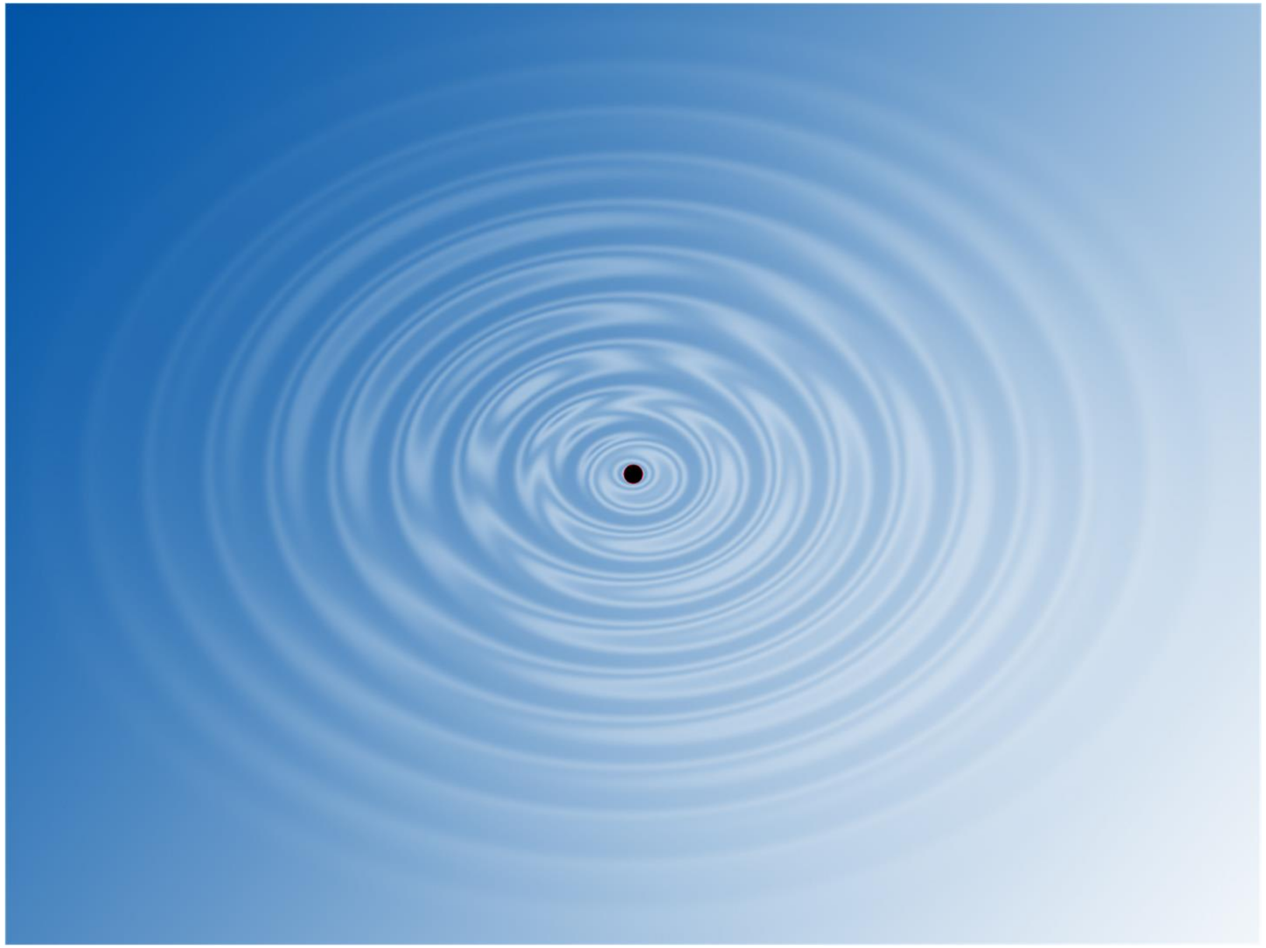
Maximilian Bosch bosch@ief.ee.ethz.ch

COMSOL: Last Time – Point Source Field



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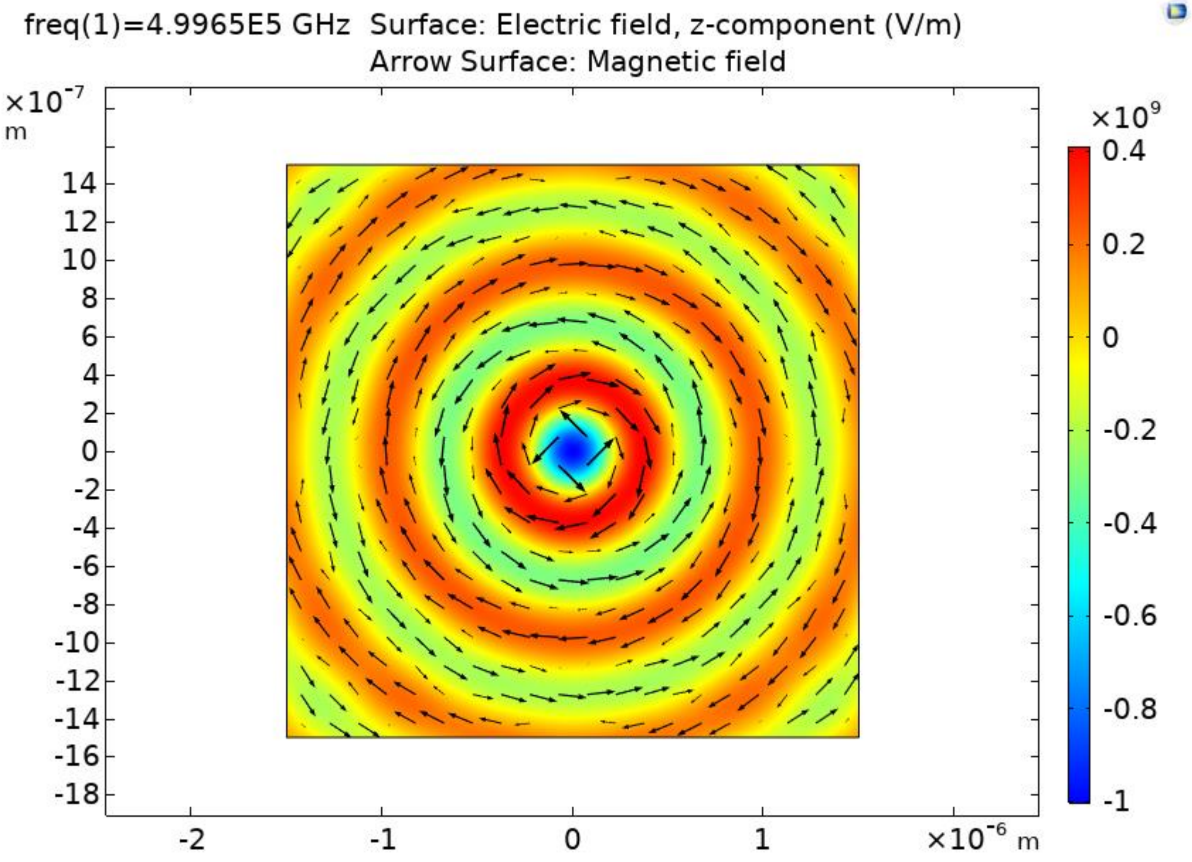
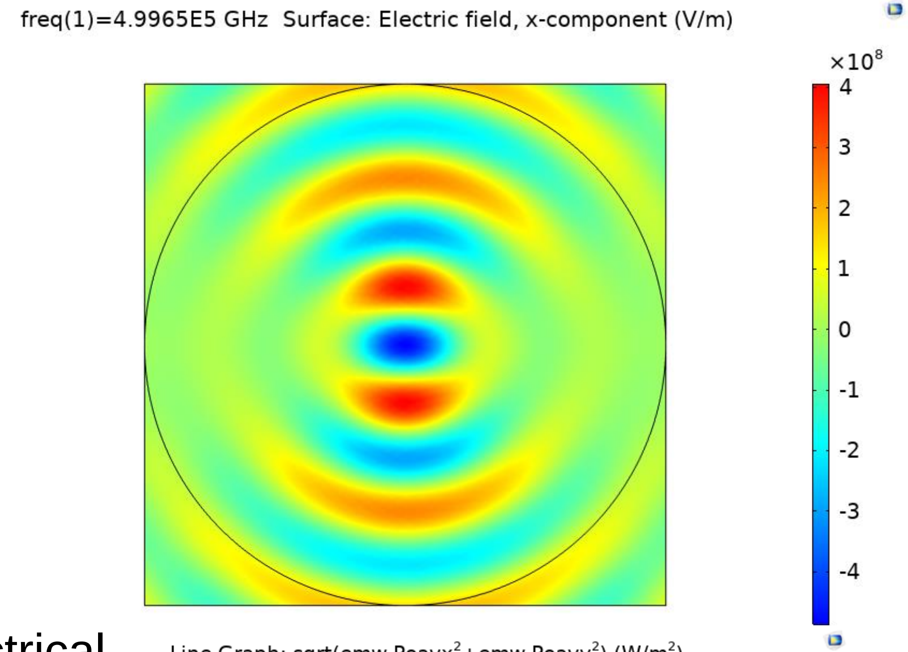
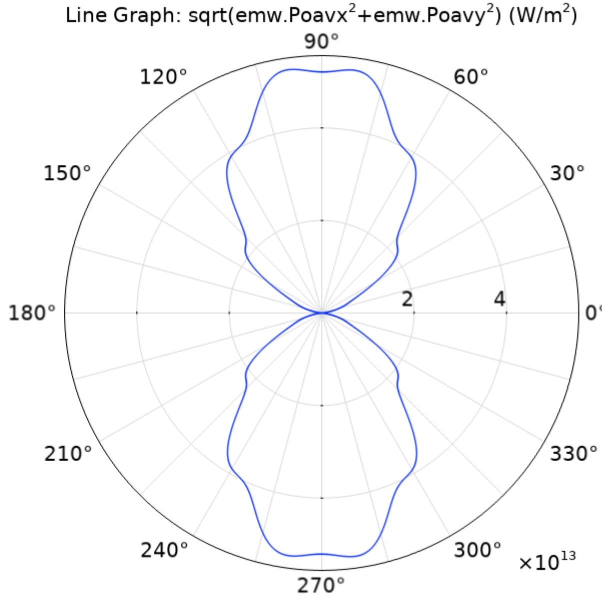
COMSOL: Last Time – Point Source Field



Using line current



Using electrical dipole



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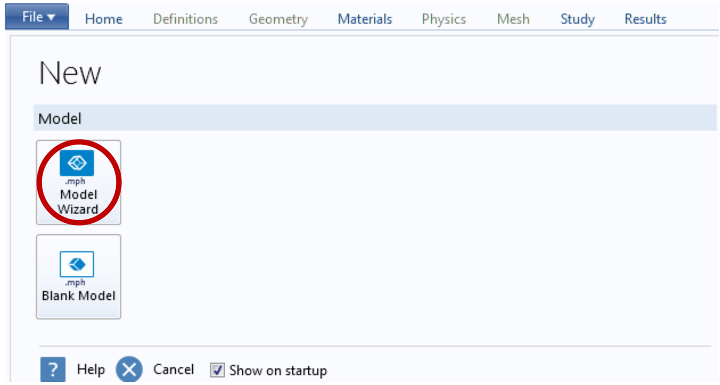
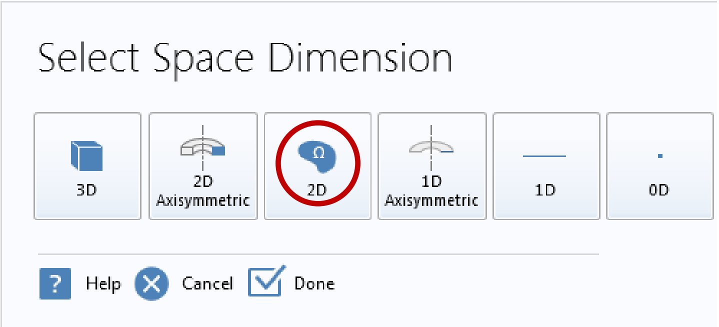
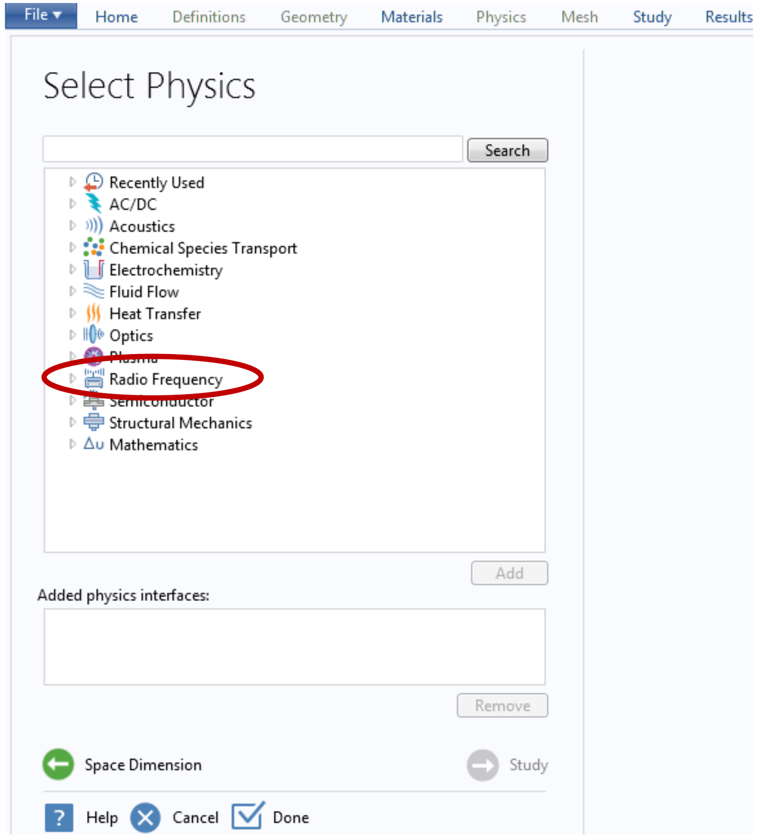
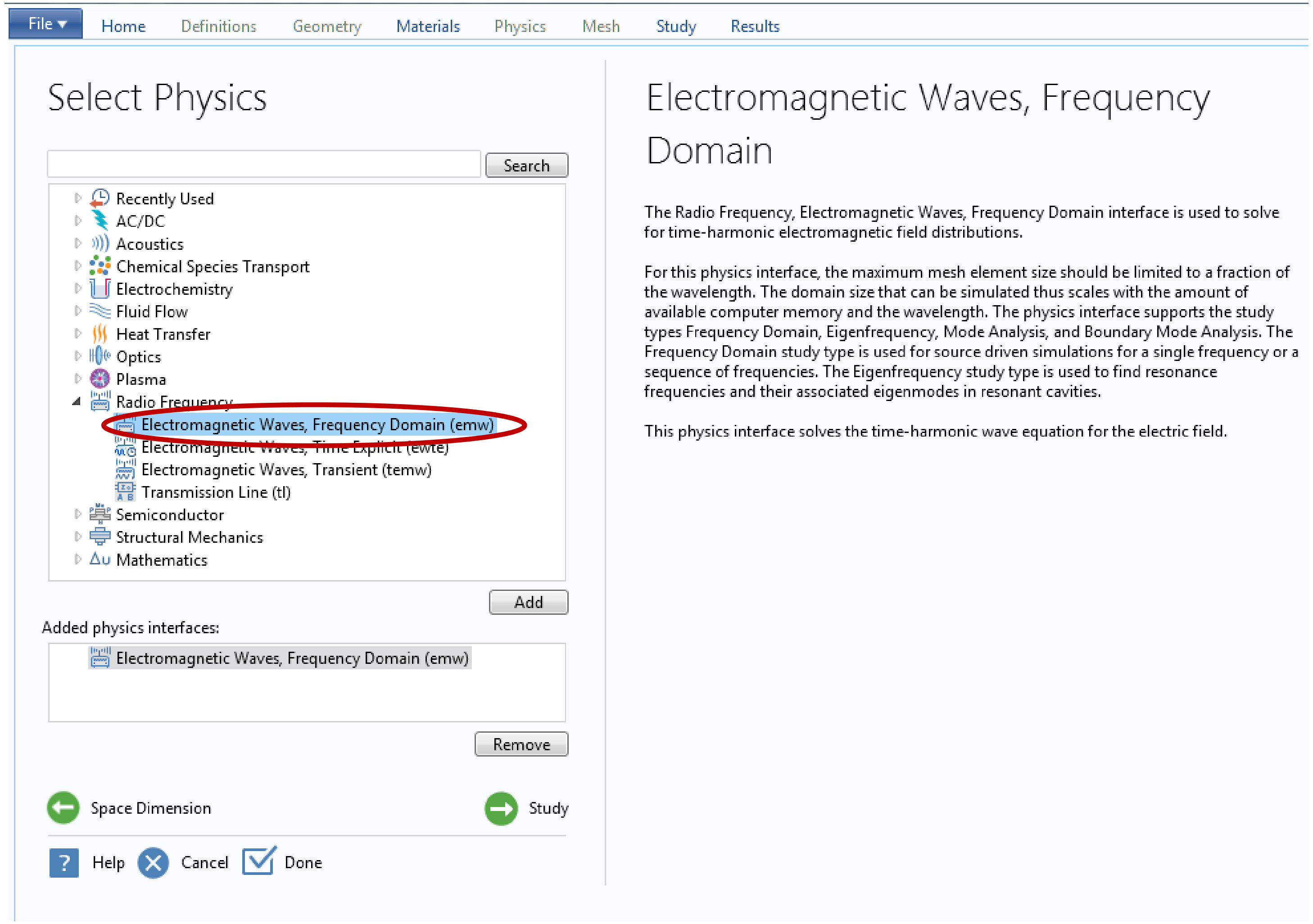
COMSOL: Last Time



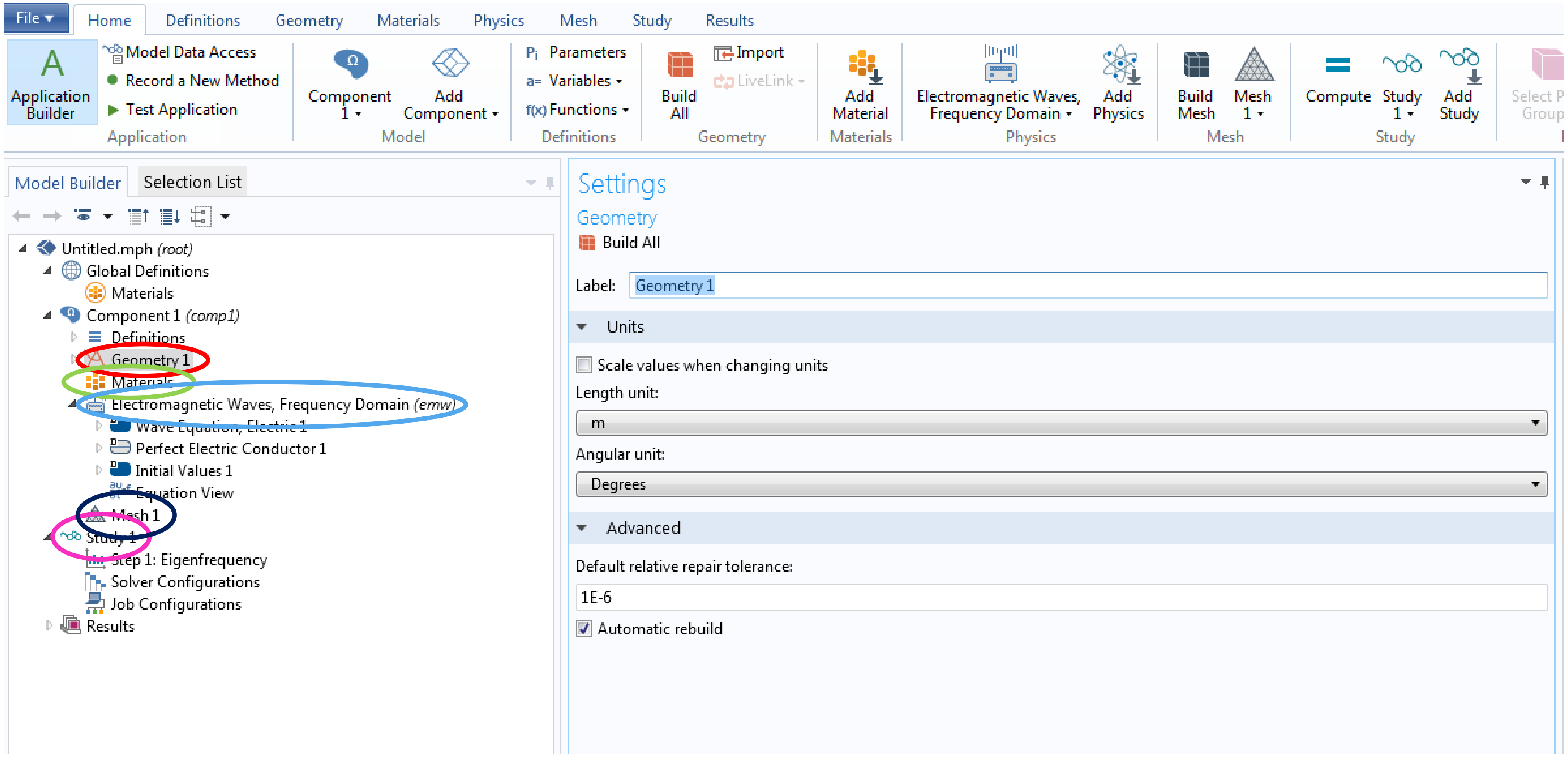




COMSOL: Last Time



COMSOL: Last Time



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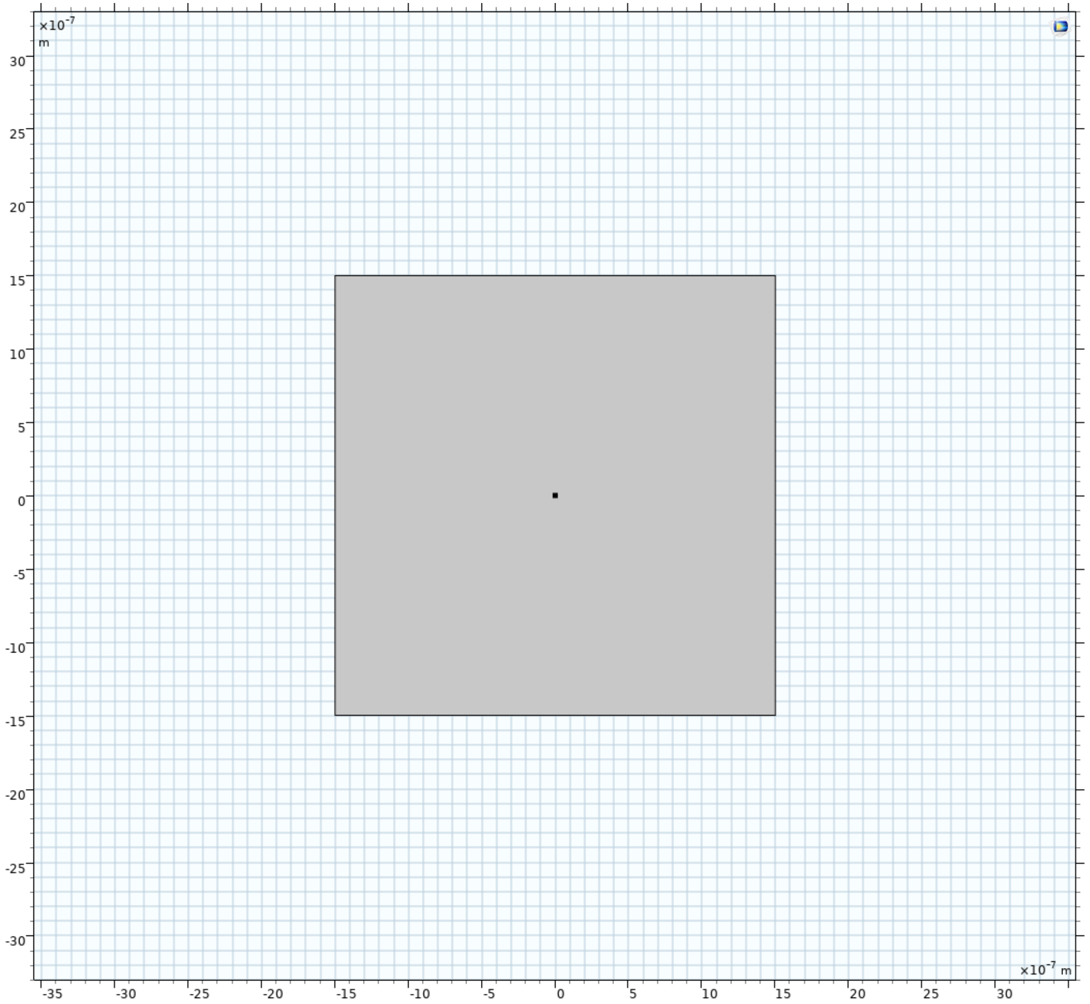
COMSOL: Last Time

- Define simulation domain
- Build geometry



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COMSOL: Last Time

- Discretization of simulation domain
  - Build mesh
- Mesh size determines accuracy of solution
  - Too large mesh ⇒ wrong results
- Accuracy vs. simulation time
  - **Today**: Optimization by manual refinement

- Discretization of simulation domain
  - Build mesh
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  - Too large mesh ⇒ wrong results
- Accuracy vs. simulation time
  - **Today**: Optimization by manual refinement

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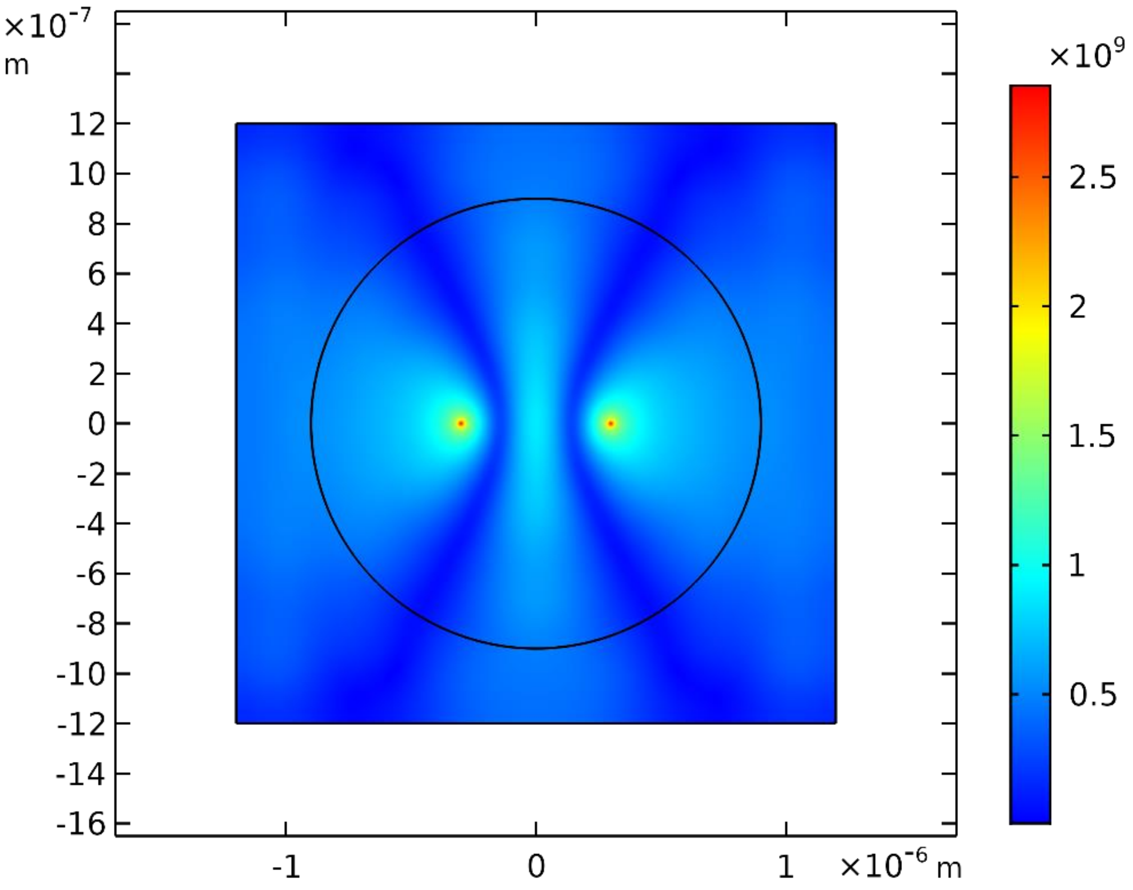
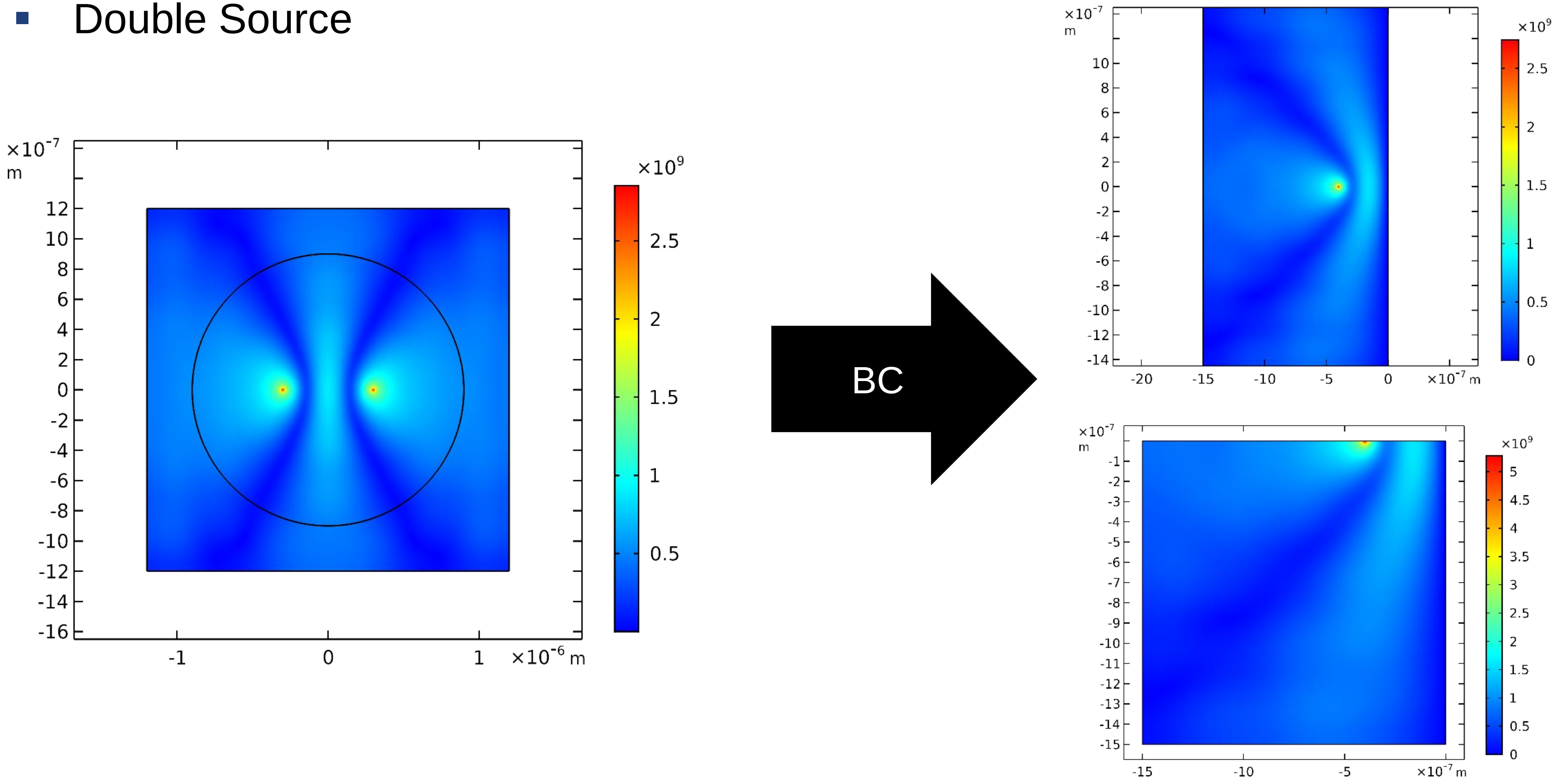
COMSOL: Today

- Mirror effect by using boundary conditions
- Double Source



COMSOL: Today

- Mirror effect by using boundary conditions
- Double Source



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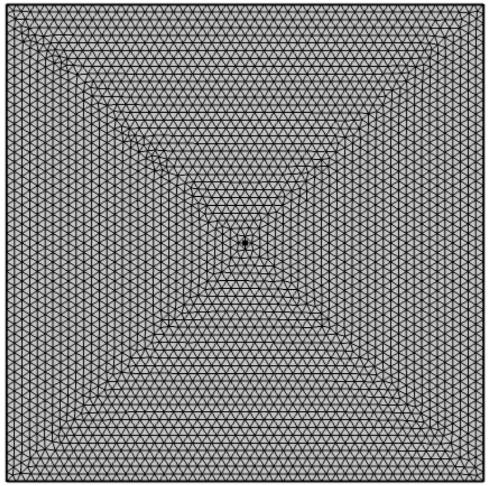
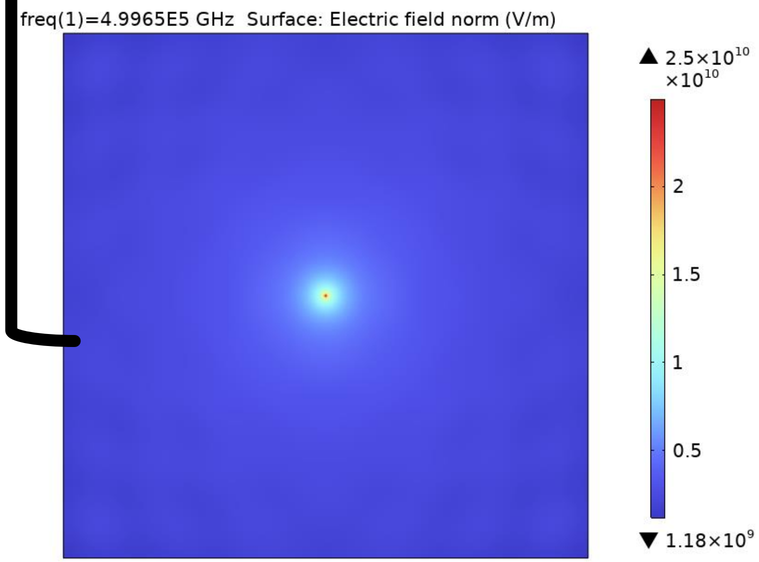
COMSOL: Today

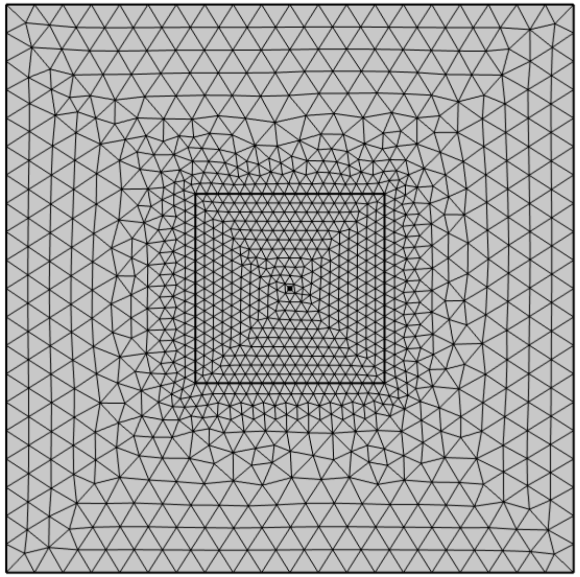
- Accuracy vs computation time
  - Too coarse mesh: incorrect results
  - Too fine mesh: unecessary large computation time

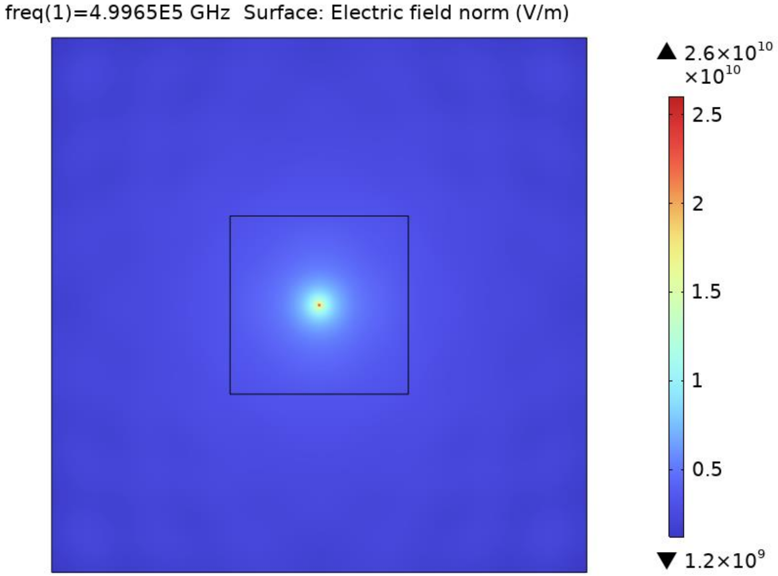
- Applying manual refinement
  - Refining **only in the region of interest**



~ 10s







~ 4s

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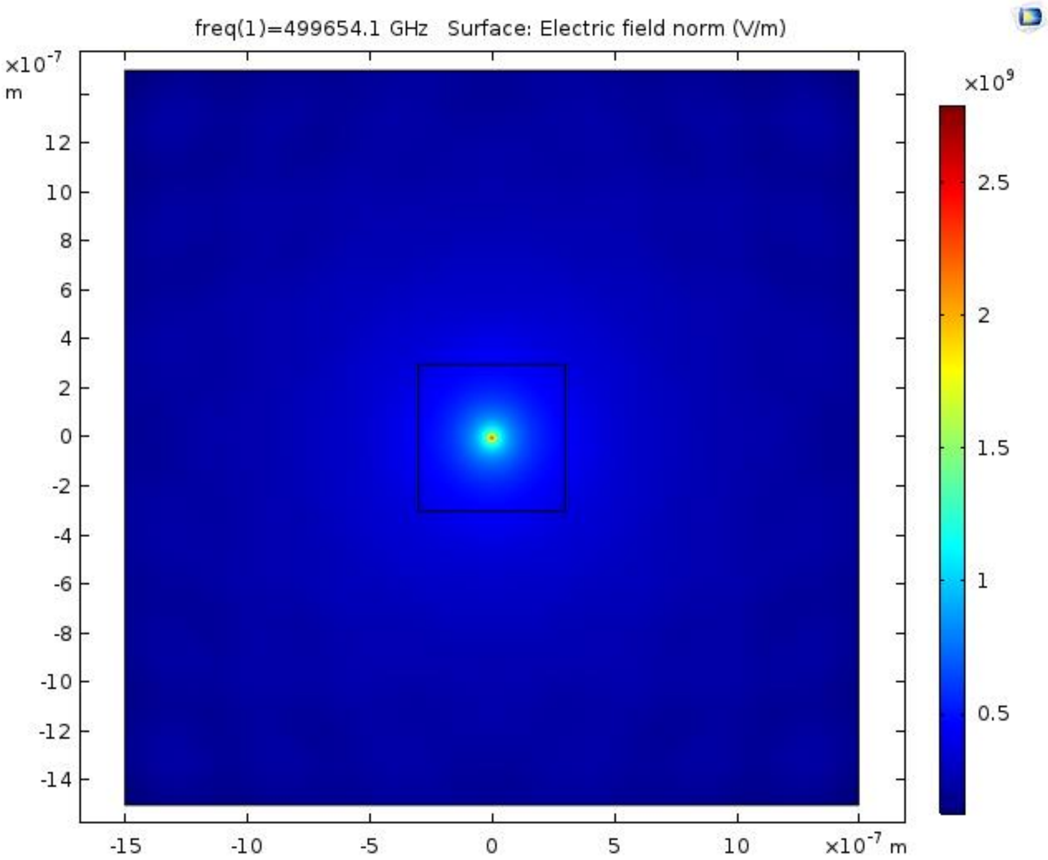
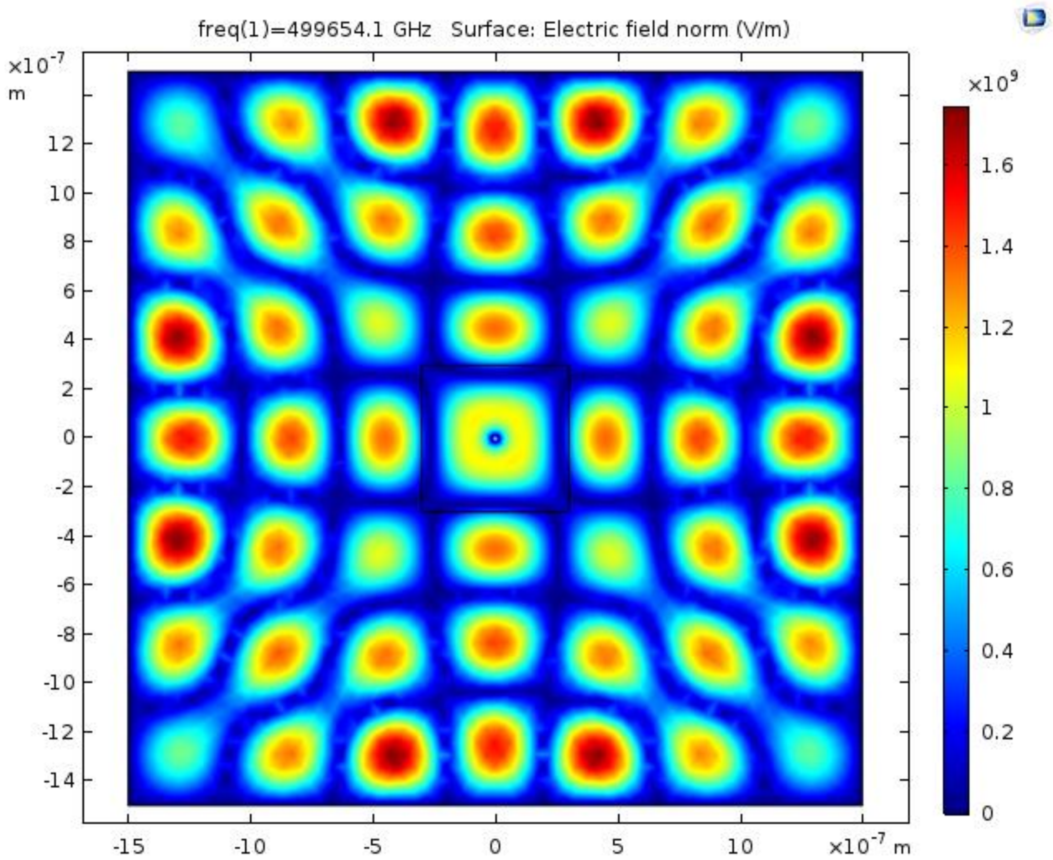
COMSOL: Last Time



Scattering Boundaries



PEC Boundaries

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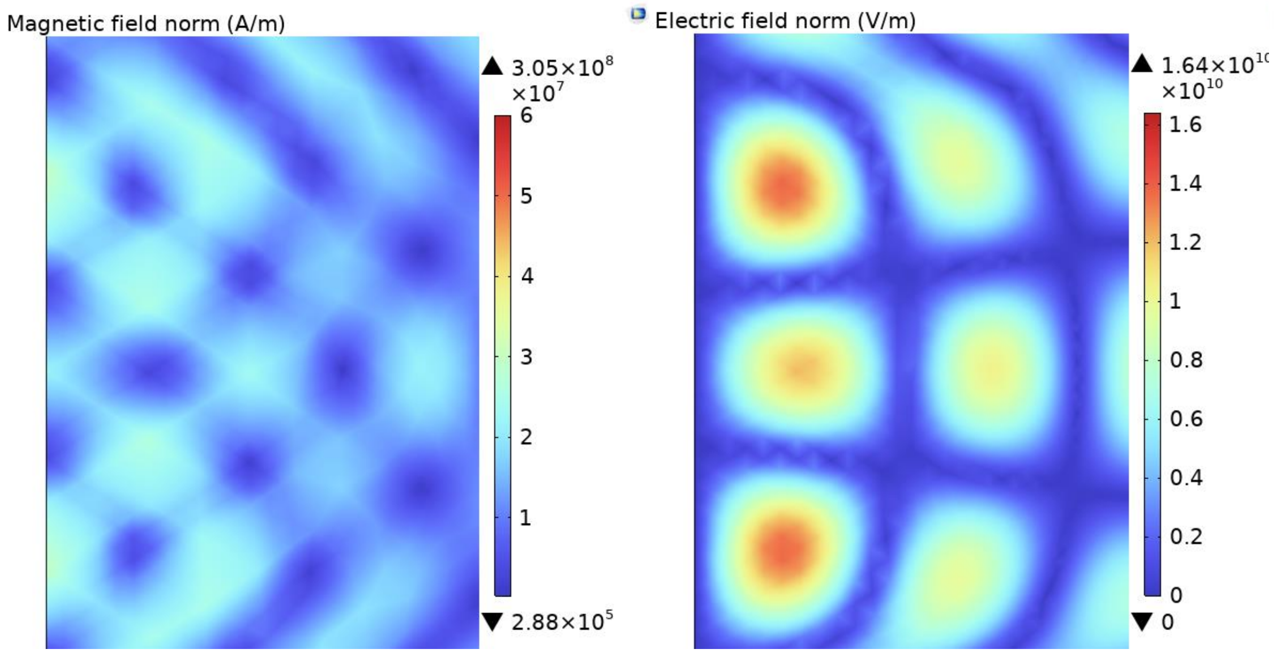
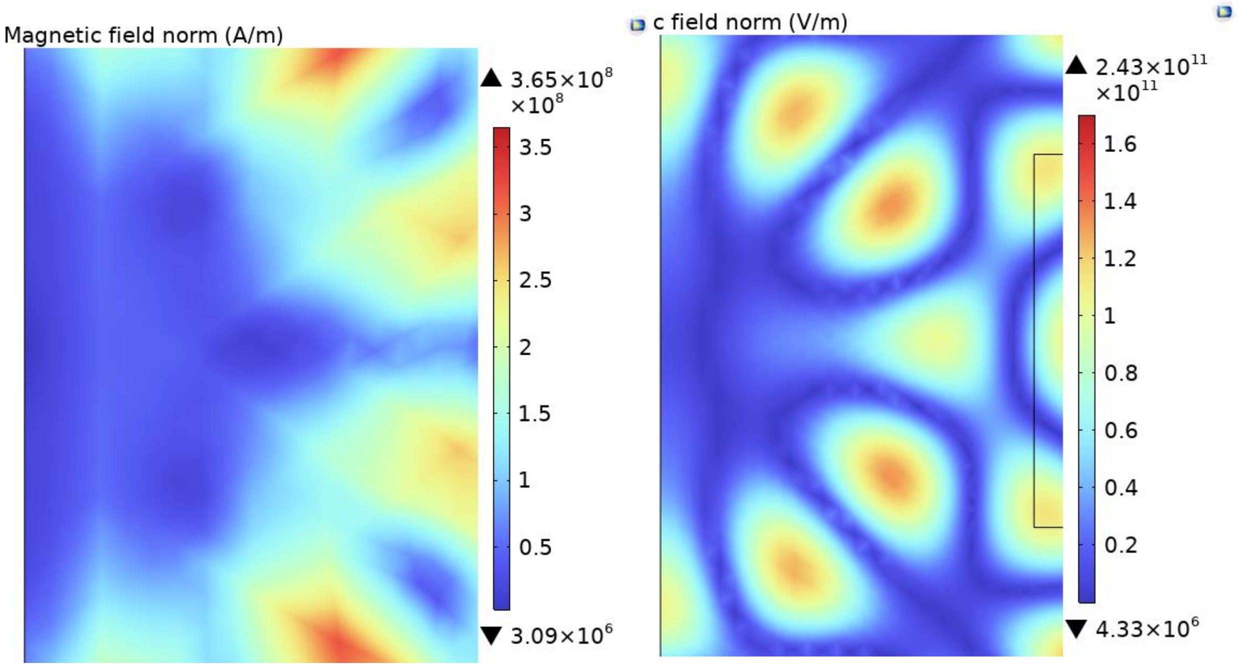
COMSOL: Today

- Distinction between PEC and PMC



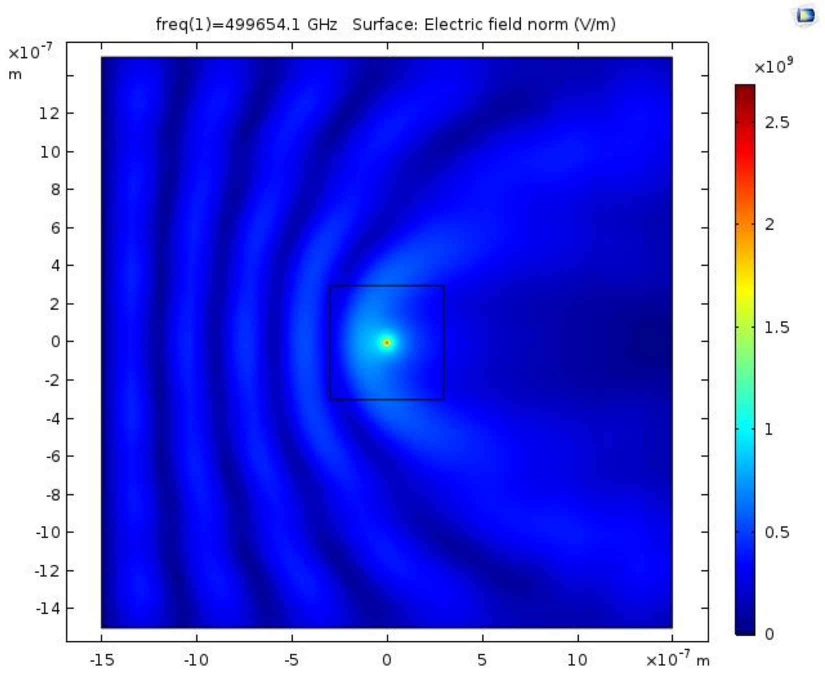


PEC
$\boldsymbol{E} = \boldsymbol{0}, \boldsymbol{H} \neq \boldsymbol{0}$ at boundaries

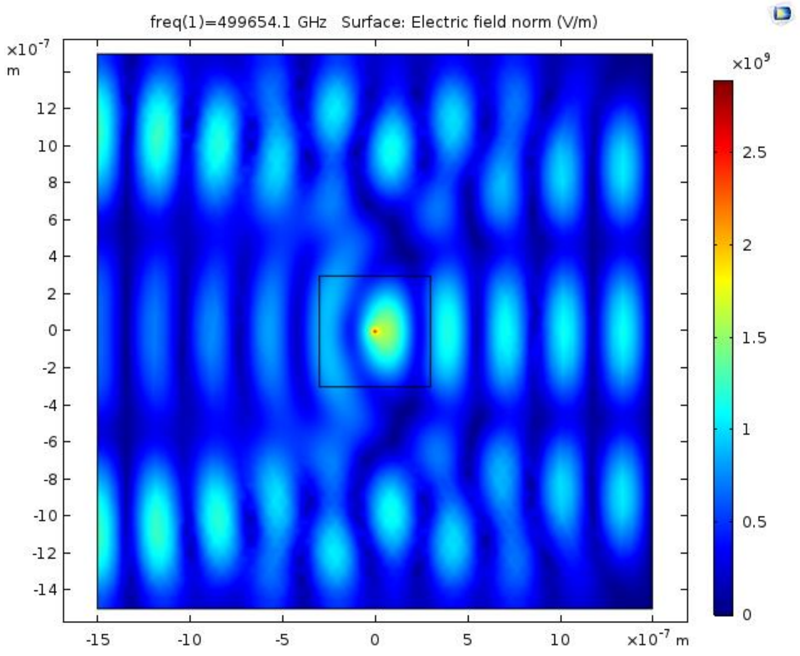
PMC
$\boldsymbol{H} = \boldsymbol{0}, \boldsymbol{E} \neq \boldsymbol{0}$ at boundaries

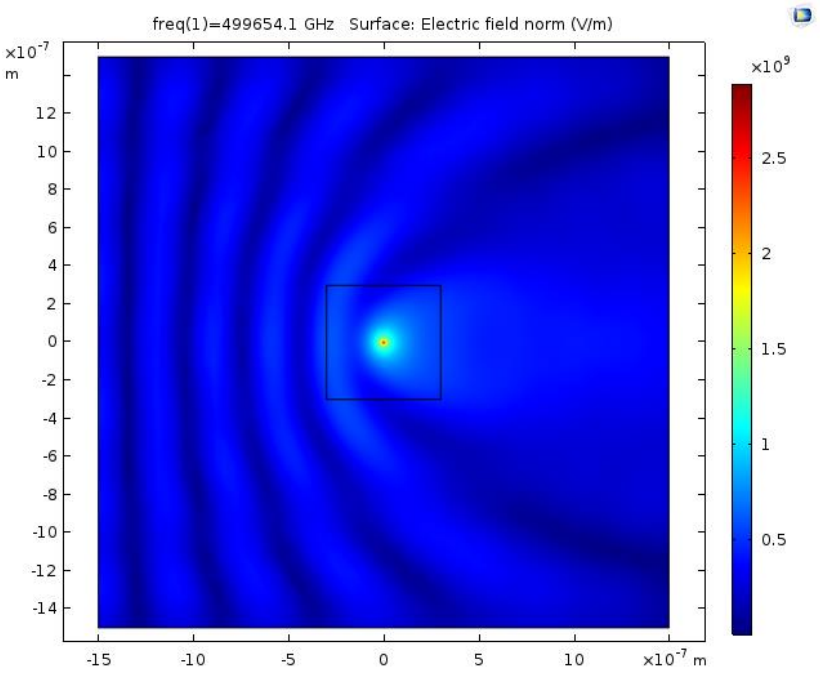
COMSOL: Today

- Mirror effect by using different boundary conditions



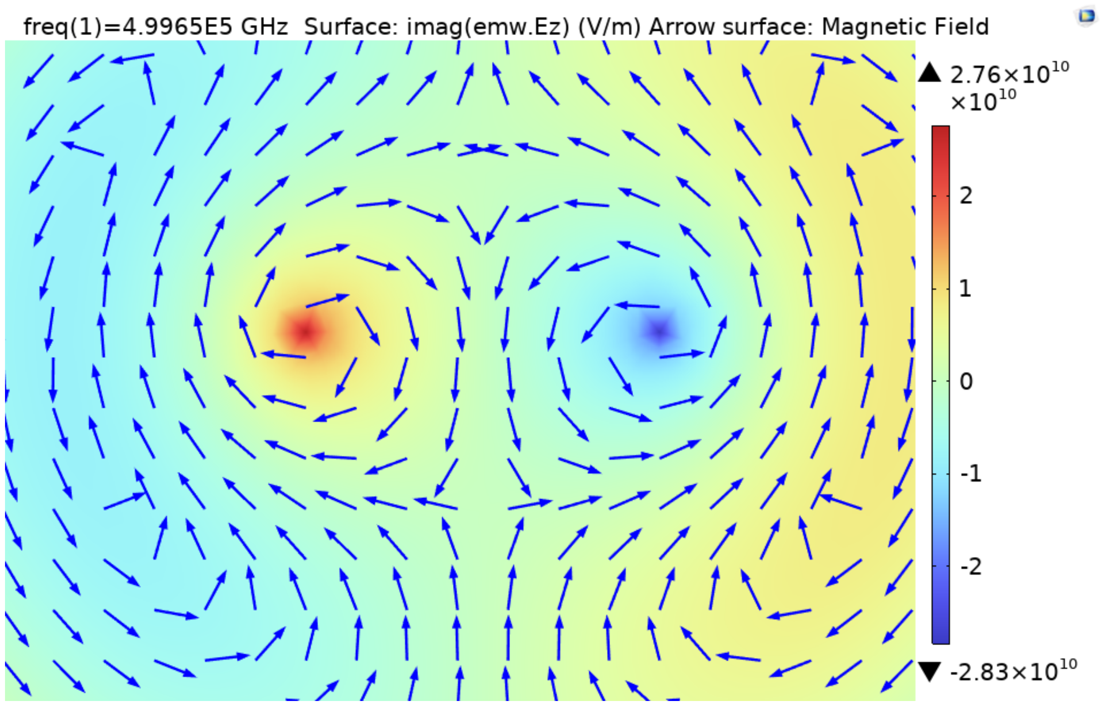
Can you see the difference?
Can you guess the boundary conditions?



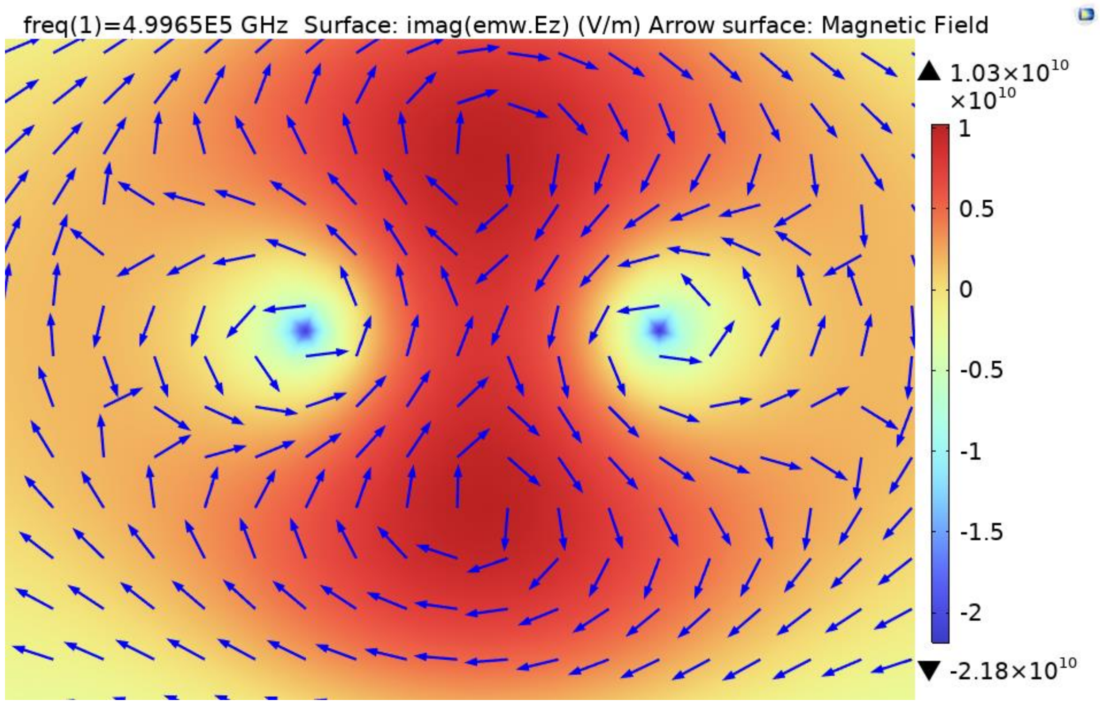


COMSOL: Today

- Double Source



With two line currents in opposite direction



With two line currents in same direction

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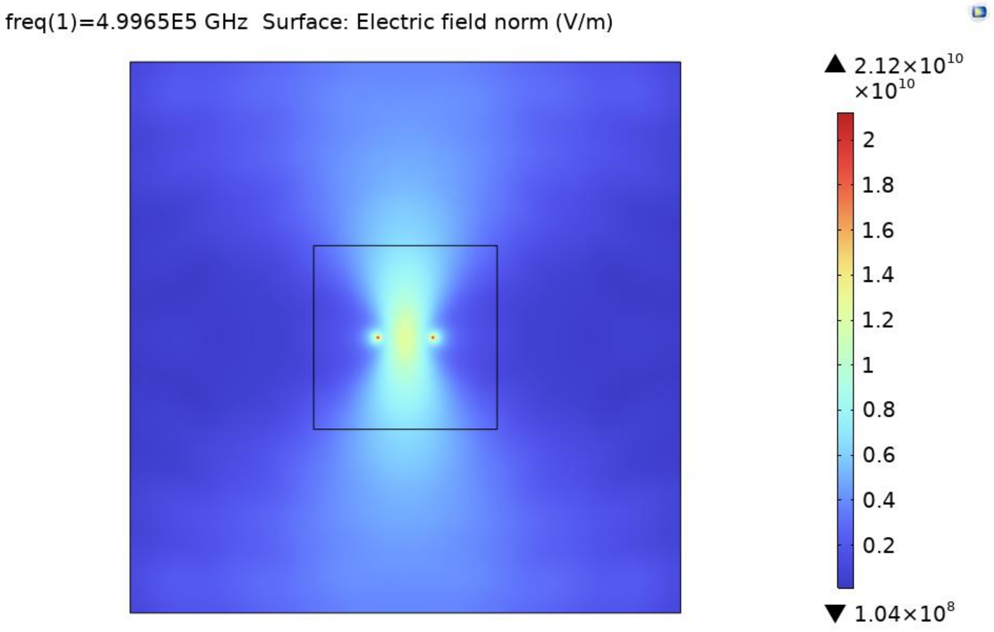
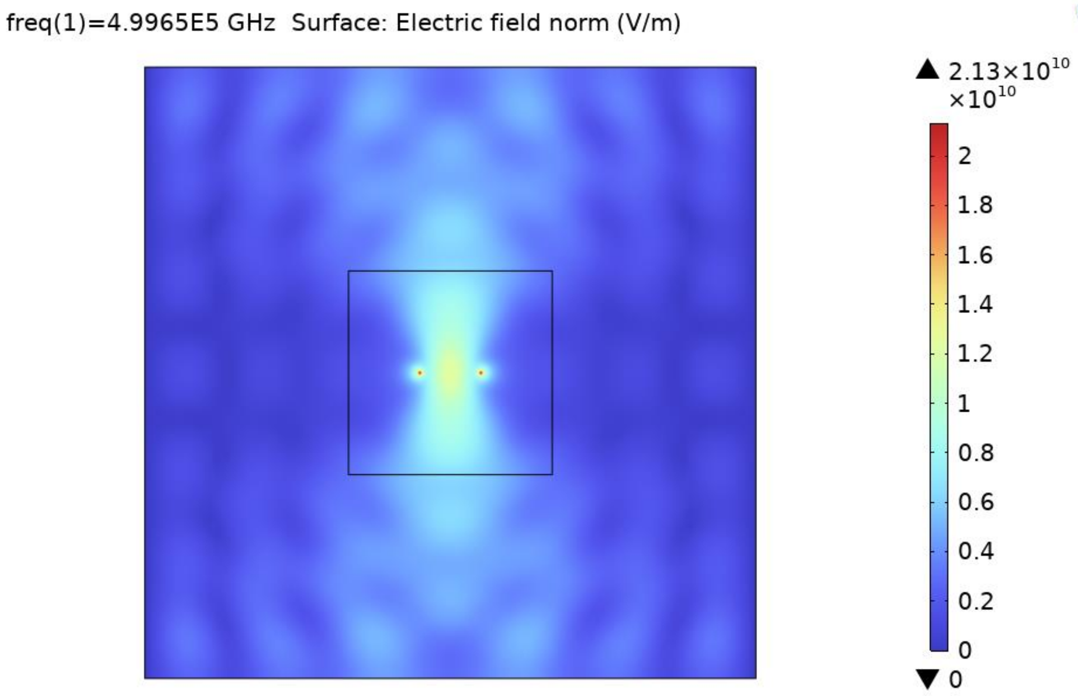
COMSOL: Today

- Effects of BCs
  - PECs behave as a mirror interface



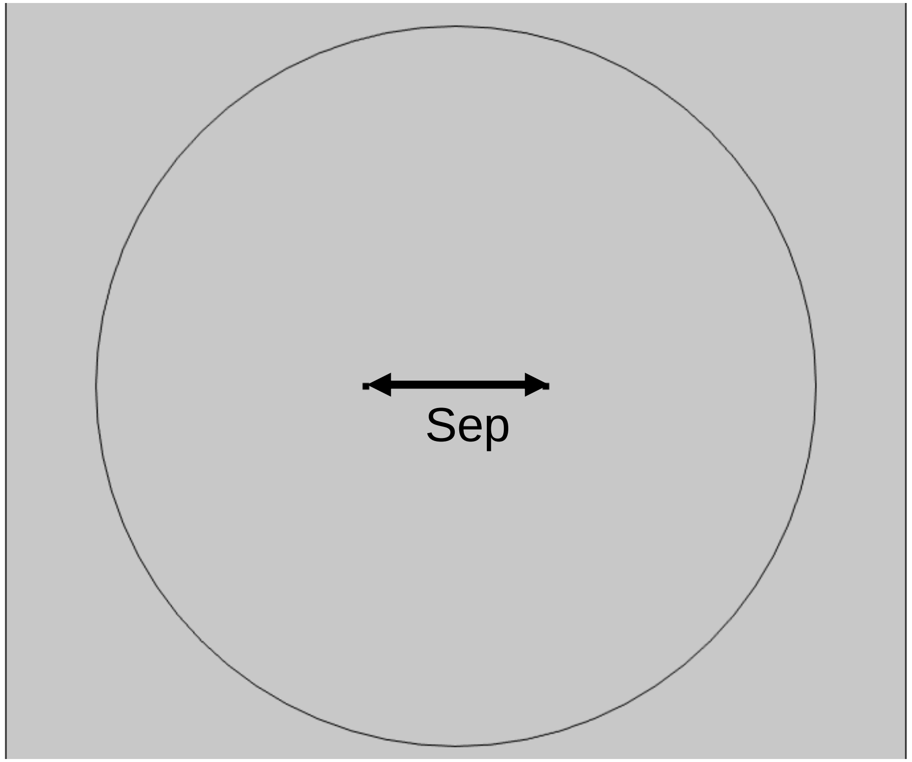
SBCs everywhere

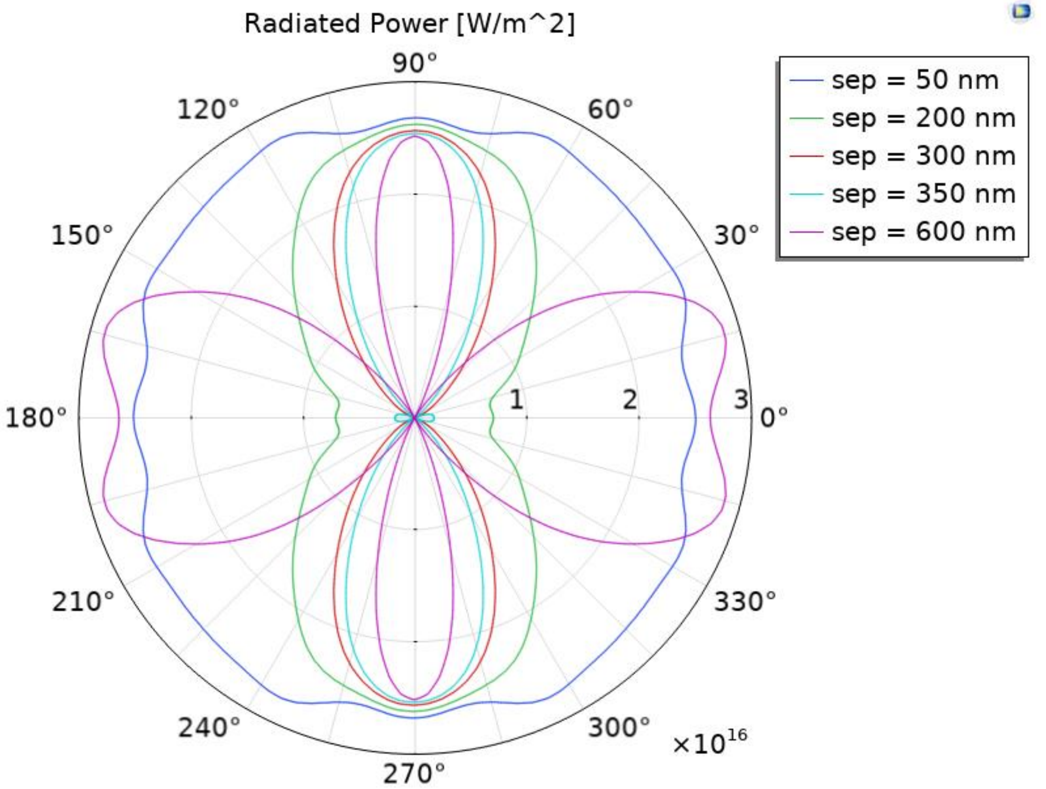


PECs at which boundary?

COMSOL: Today

- Optimizing a parameter → parameter sweep
  - It allows us to evaluate our model's properties w.r.t this parameter





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