



P&S COMSOL® Design Tool

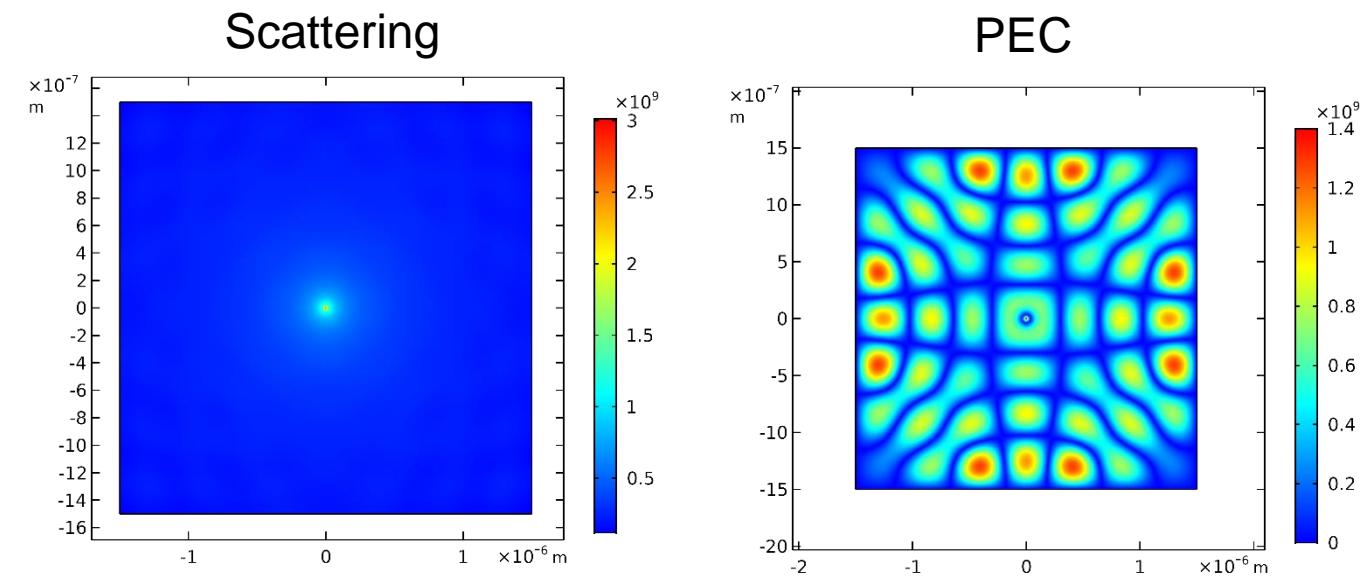
Week 2: EM Introduction & Introduction to COMSOL

Taichiro Fukui, Maximilian Bosch

Recap

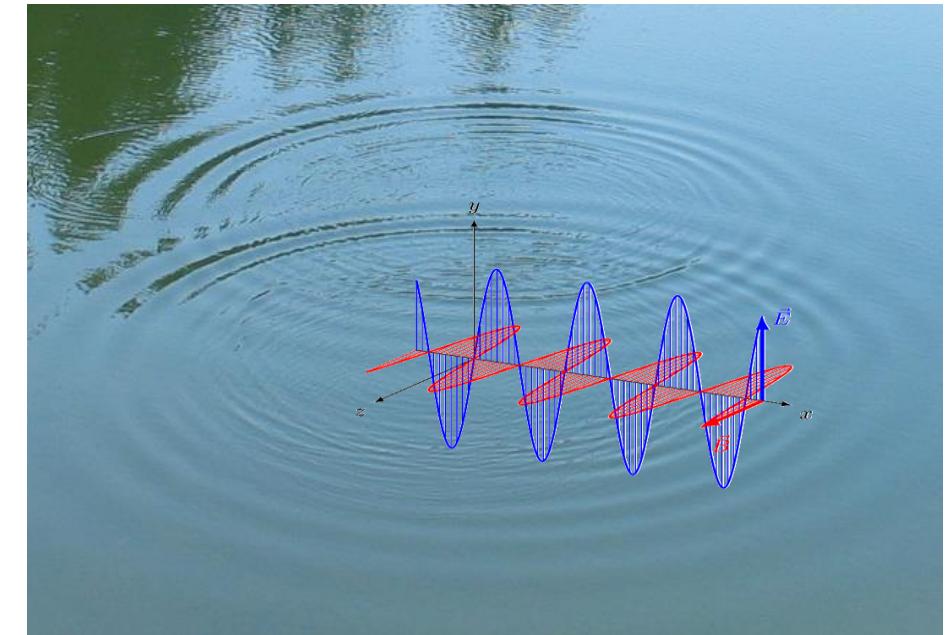
- Last Time
 - Introduction to COMSOL
 - Point Source model
 - Scattering Boundary Conditions
 - Perfect Electric Conduction (PEC)

- Today
 - More on Boundary Conditions
 - Perfect Electric Conductor (PEC)
 - Perfect Magnetic Conductor (PMC)
 - Scattering Boundary Condition
 - Periodic Boundary Condition (PBC)
 - Perfectly Matched Layer (PML)



Motivation

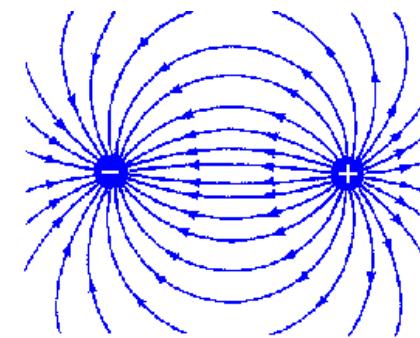
- Wave behavior in nature
- But electromagnetic waves are (usually) invisible
- → COMSOL visualizes them!
- → Intuitive approach to understand mathematics
- Mathematics will be treated extensively in EM lectures



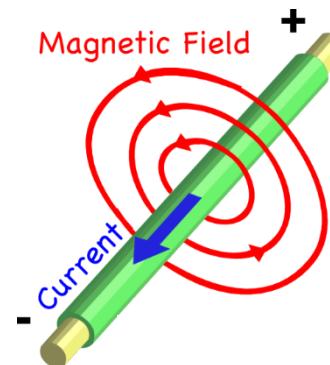
Electromagnetics: What you've seen before

- Physical phenomena and laws

Electric field between positive and negative charge

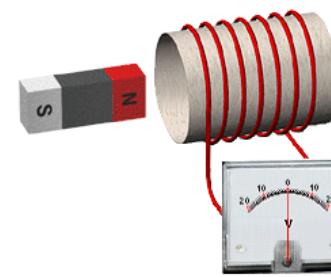


Current flows
→ magnetic field



Faraday's law of induction

$$u(t) = -\frac{d\Phi}{dt} = -\frac{d}{dt} \iint_A \vec{B} \cdot d\vec{A}$$



Coulomb force: $F = Eq$

Ohmic law: $U = R \cdot I$

Kirchhoff current & voltage law:

$$\sum_{node} I_i = 0, \sum_{loop} U_i = 0$$

Electromagnetics: Maxwell's Equations (local form)

- Physical phenomena and laws

Gauss's law:

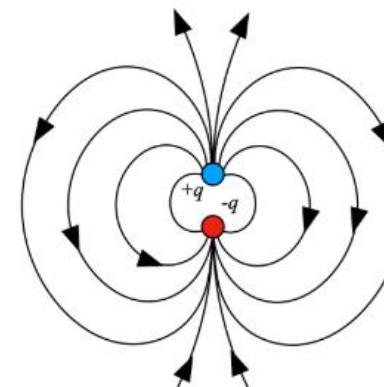
$$\nabla \cdot D = \rho$$

Electric displacement Field D
(Free) charge density ρ

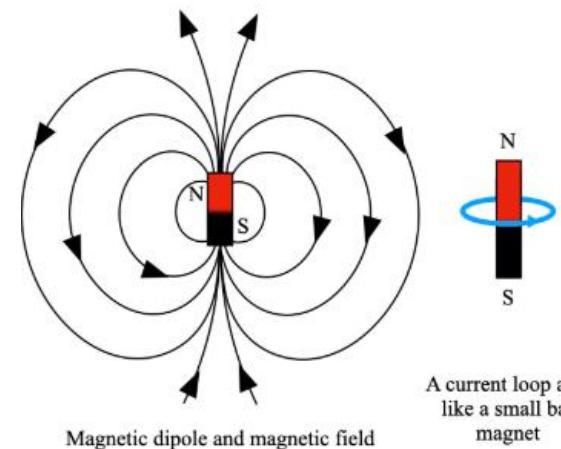
Gauss's law (Magnetism):

$$\nabla \cdot B = 0$$

Magnetic Flux Density B



Electric dipole and electric field

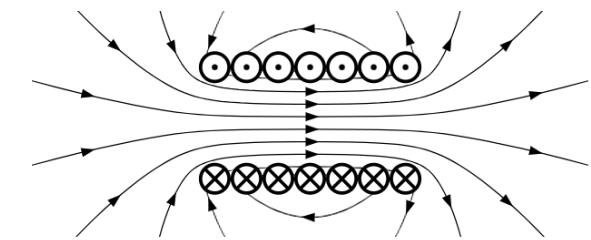


A current loop acts
like a small bar
magnet

Faraday's law:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Electric Field E



Ampère's law:

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

(Free) current density J
Magnetic Field H

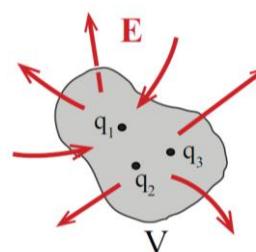
Electromagnetics: Maxwell's Equations (global form)

- Physical phenomena and laws By using $\int_V \nabla \cdot \mathbf{F} dV = \oint_{\partial V} \mathbf{F} \cdot d\mathbf{A}$ and $\int_A (\nabla \times \mathbf{F}) \cdot d\mathbf{A} = \oint_{\partial A} \mathbf{F} \cdot d\mathbf{l}$

Gauss's law:

$$\oint_{\partial V} \mathbf{D} \cdot d\mathbf{A} = \int_V \rho dV$$

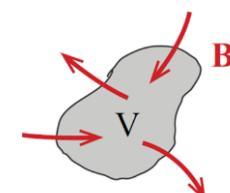
Electric displacement Field D
(Free) charge density ρ



Gauss's law (Magnetism):

$$\nabla \cdot \mathbf{B} = 0 \Leftrightarrow \oint_{\partial V} \mathbf{B} \cdot d\mathbf{A} = 0$$

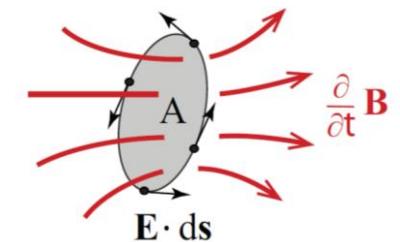
Magnetic Flux Density B



Faraday's law:

$$\oint_{\partial A} \mathbf{E} \cdot d\mathbf{s} = -\frac{\partial}{\partial t} \int_A \mathbf{B} \cdot d\mathbf{A}$$

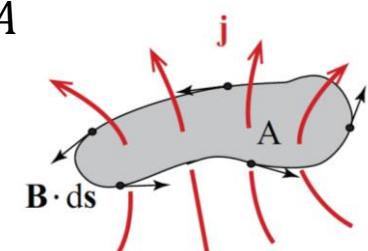
Electric Field H



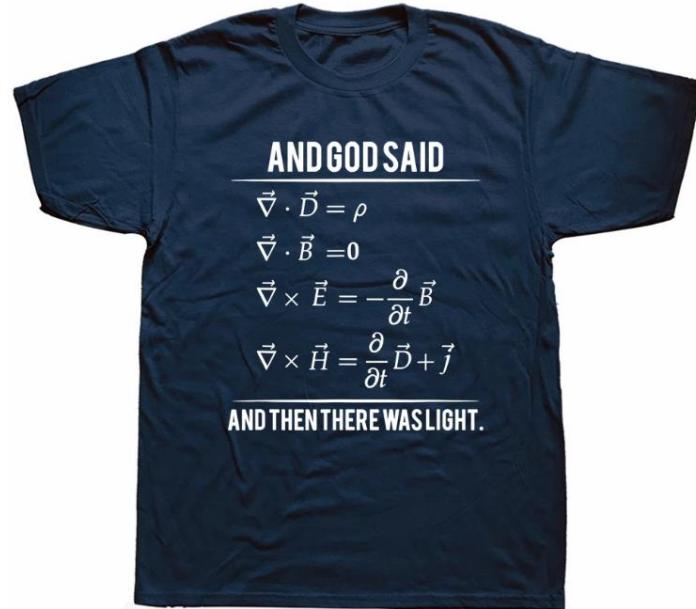
Ampère's law:

$$\oint_{\partial A} \mathbf{H} \cdot d\mathbf{l} = \int_A \left(\mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{A}$$

(Free) current density J
Magnetic Field H



Electromagnetics: Maxwell's Equations



- How do these equations describe light propagation in materials?
- One needs a model that captures the impact of fields on the material

- Electric Case **Polarization P**

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}(E) \approx \epsilon_0 \vec{E} + \epsilon_0 \chi_E^{(1)} \vec{E}$$

- Magnetic Case **Magnetization M**

$$\vec{B} = \mu_0 \vec{H} + \vec{M}(H) \approx \mu_0 \vec{H} + \mu_0 \chi_H^{(1)} \vec{H}$$

$$\begin{aligned}\epsilon_r &= 1 + \chi_E^{(1)} \\ \mu_r &= 1 + \chi_M^{(1)}\end{aligned}\quad \text{In linear case}$$

Nonlinear Optics: 227-0655-00L

Electromagnetics: Materials Relations

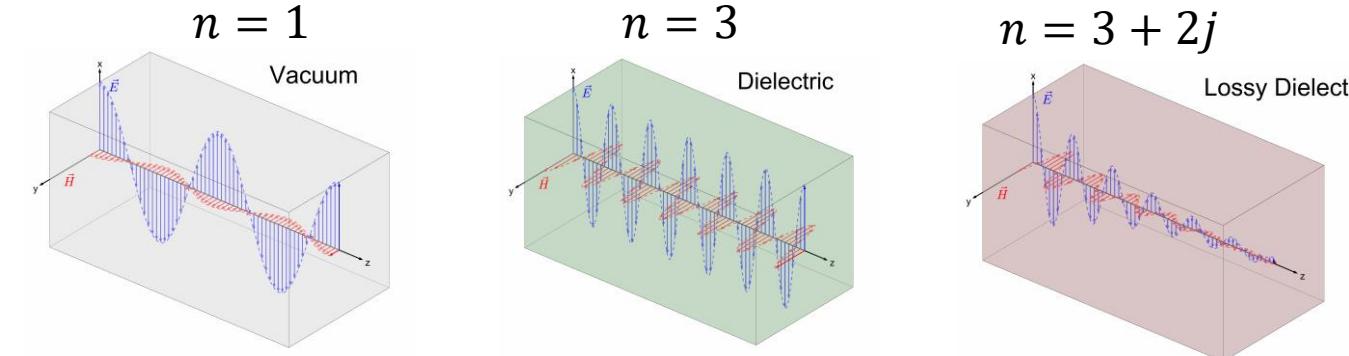
- To analyze EM problems, we need to define the **material properties** involved
- Materials are defined by their **refractive index n** which is defined as
 - $n = \sqrt{\mu_r \epsilon_r}$, for vacuum $n = 1$
 - n is a complex number $n = n' + jk$

$$n = n' + jk$$

Influences wavelength Influences losses

$$\epsilon_r = 1 + \chi_E^{(1)}$$

$$\mu_r = 1 + \chi_M^{(1)}$$



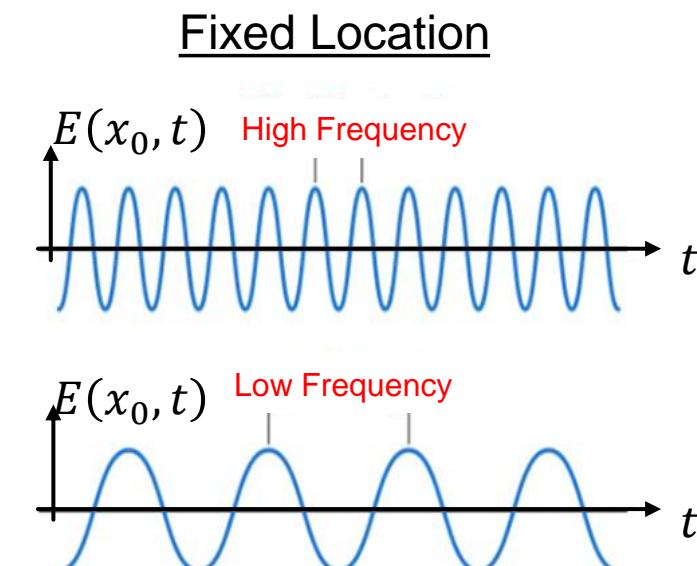
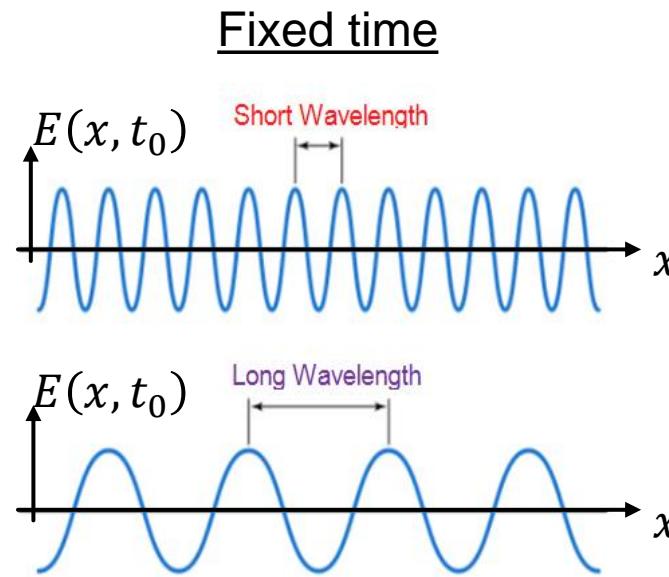
Electromagnetics: Maxwell's Equations

- Wave Equation from Maxwell's Equations
 - Homogenous, Isotropic, linear Material,
 - No Sources

$$\frac{\partial^2 E_z}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2} = 0$$

- General Solution

$$E_z(x, t) = g(ct \pm x)$$



Electromagnetics: Wavelength and Frequency

- Plane wave Solution:

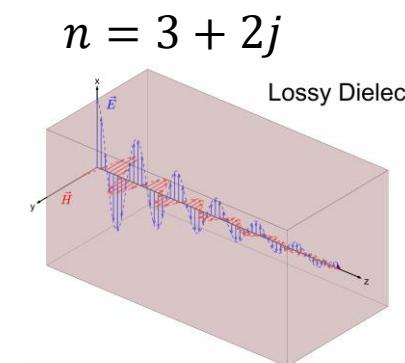
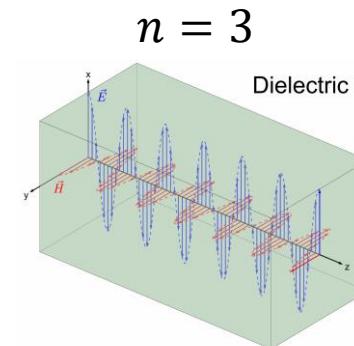
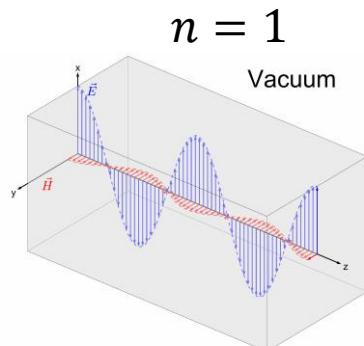
$$E_z(x, t) = E_0 e^{j\omega t} e^{-jkx}$$

- Proof:

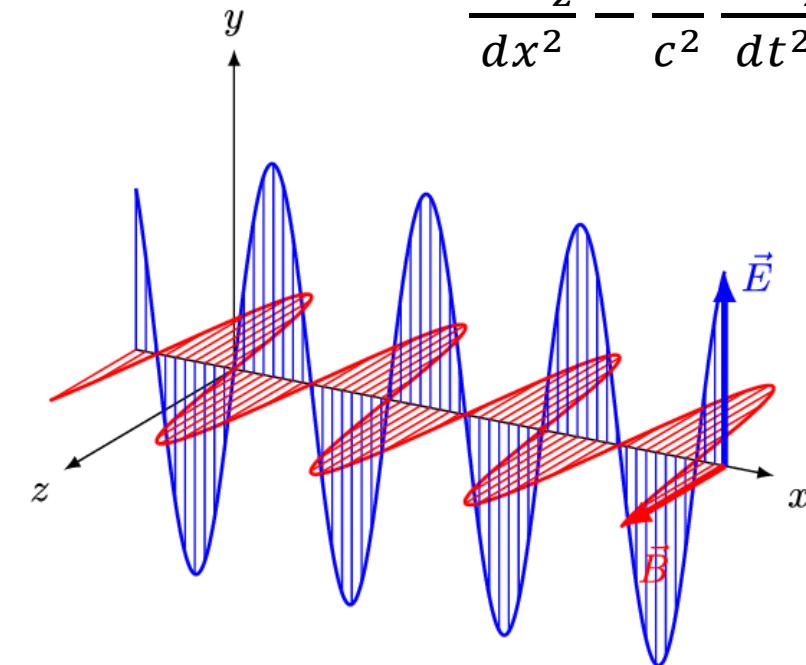
$$-k^2 e^{j\omega t} e^{-jkx} + \frac{\omega^2}{c^2} e^{j\omega t} e^{-jkx} = 0$$

- With

$$k^2 = \frac{\omega^2}{c^2} = \frac{\omega^2}{c_0^2} n^2$$



$$\frac{\partial^2 E_z}{dx^2} - \frac{1}{c^2} \frac{\partial^2 E_z}{dt^2} = 0$$



And what does Comsol solve?

The wave equation: $\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2 \left(\epsilon_r - \frac{j\sigma}{\omega \epsilon_0} \right) \mathbf{E} = \mathbf{0}$

First step: Insert magnetization into the Faraday's equation $\nabla \times \mathbf{E} = -\mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t} \Leftrightarrow \mu_r^{-1}(\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$

Second step: Apply the curl's operation $\nabla \times (\mu_r^{-1}(\nabla \times \mathbf{E})) = \nabla \times \left(-\mu_0 \frac{\partial \mathbf{H}}{\partial t} \right) = -\mu_0 \nabla \times \left(\frac{\partial \mathbf{H}}{\partial t} \right) = -\mu_0 \frac{\partial}{\partial t} (\nabla \times \mathbf{H})$

Third step: Insert local Ohm's law and polarization into Ampere's equation

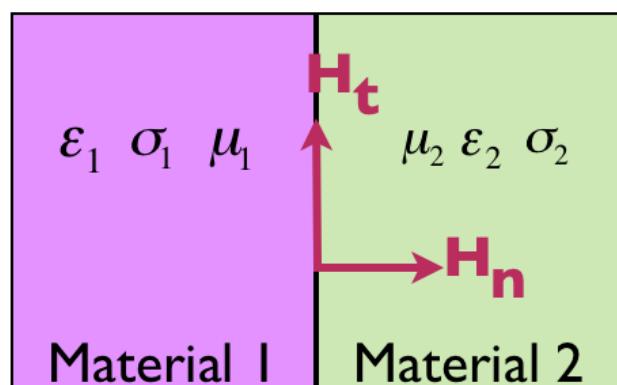
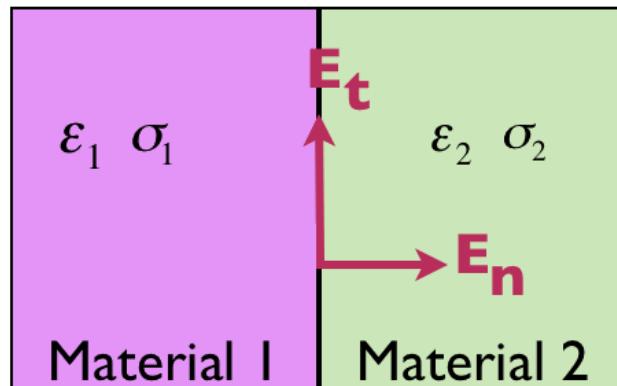
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \sigma \mathbf{E} + \epsilon_0 \epsilon_r \frac{\partial \mathbf{E}}{\partial t} \Leftrightarrow \nabla \times (\mu_r^{-1}(\nabla \times \mathbf{E})) = -\mu_0 \frac{\partial}{\partial t} \left(\sigma \mathbf{E} + \epsilon_0 \epsilon_r \frac{\partial \mathbf{E}}{\partial t} \right) = -\mu_0 \sigma \frac{\partial \mathbf{E}}{\partial t} - \mu_0 \epsilon_0 \epsilon_r \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Fourth step: Solve the equation into the frequency domain: $\frac{\partial}{\partial t} \leftrightarrow j\omega$

$$\nabla \times (\mu_r^{-1}(\nabla \times \mathbf{E})) = \mathbf{E} (-j\mu_0 \omega \sigma + \mu_0 \epsilon_0 \epsilon_r \omega^2) \Leftrightarrow \nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2 \left(\epsilon_r - \frac{j\sigma}{\omega \epsilon_0} \right) \mathbf{E} = \mathbf{0}$$

Boundary Conditions

- Maxwell's Boundary Conditions



Boundary Conditions

$$E_{t,1} - E_{t,2} = 0$$

$\epsilon_1 E_{n,1} - \epsilon_2 E_{n,2} = \rho_s$ (surface charge density at the interface)

$H_{t,1} - H_{t,2} = J_s$ (surface current density at the interface)

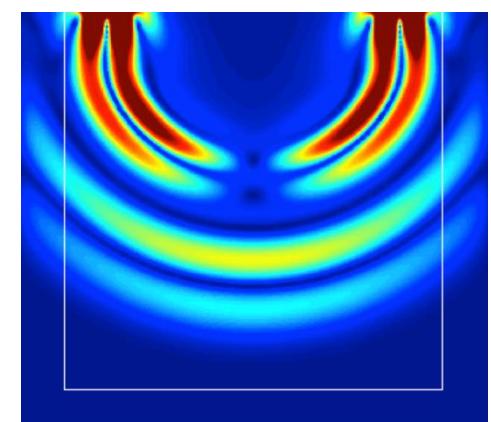
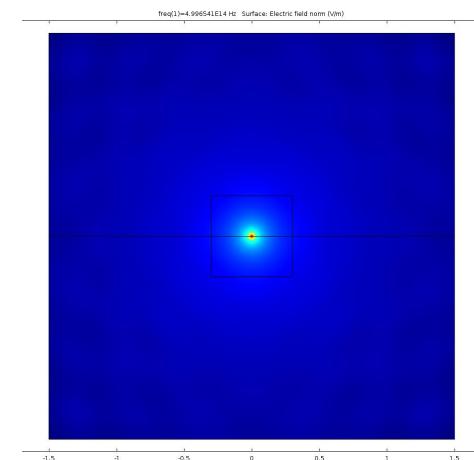
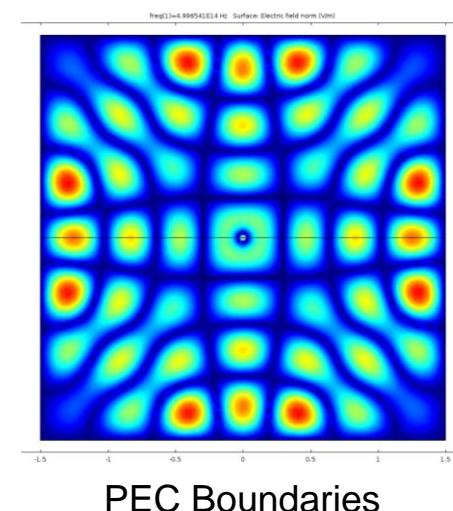
$$\mu_1 H_{n,1} - \mu_2 H_{n,2} = 0$$

227-0160-00L: Fundamentals of Physical Modeling and Simulations

227-0110-00L: Electromagnetic Waves: Materials, Effects, and
Antennas:

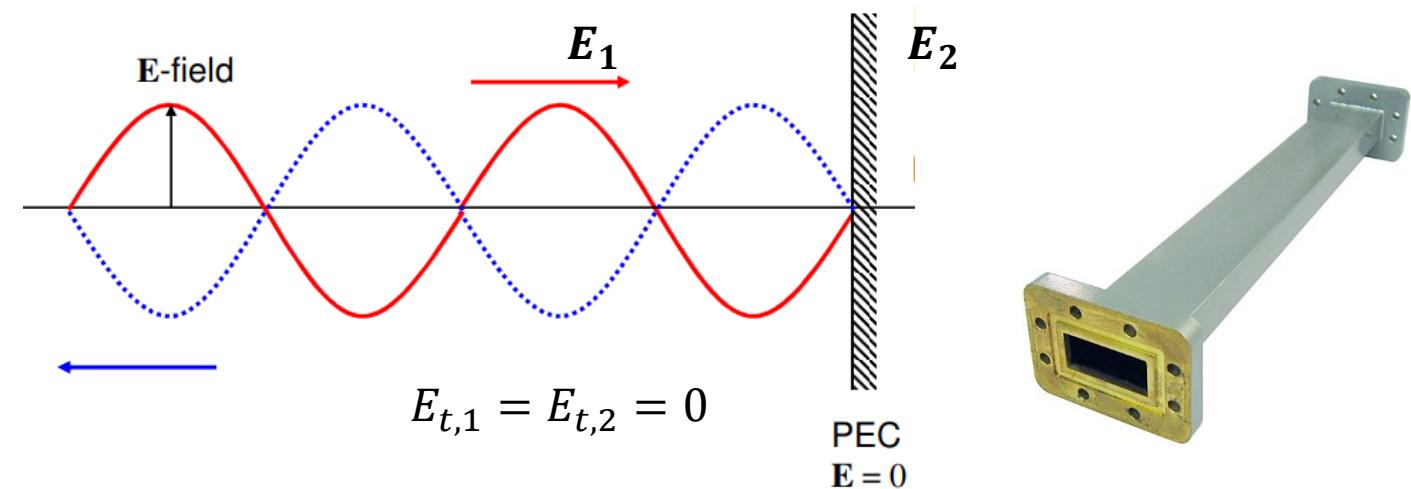
COMSOL: Boundary Conditions

- Purpose of boundary conditions → define simulation domain
- Types of boundary conditions in COMSOL
 - Perfect Electric Conductor (PEC)
 - Perfect Magnetic Conductor (PMC)
 - Scattering Boundary Condition (SBC)
 - Periodic Boundary Condition (PBC)
 - Perfectly Matched Layer (PML)



COMSOL: Boundary Conditions

- **Perfect Electric Conductor (PEC)**
 - Properties
 - Equivalent to infinite electric conductivity
 - $J = \sigma E_2 \Leftrightarrow E_2 = \frac{J}{\sigma} \xrightarrow{\sigma \rightarrow \infty} 0$: no electric outside the domain, **the E field is back reflected to the domain**



- Examples
 - Microwave planar structures
 - Metallic substrates
 - Short circuit interface
 - To implement some certain symmetry (antisymmetrical **E** field, symmetrical **H** field)

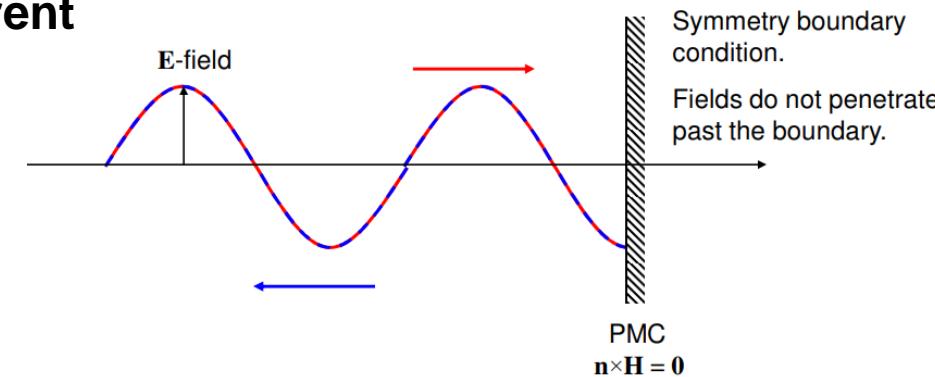
COMSOL: Boundary Conditions

- Perfect Magnetic Conductor (PMC)

- Properties

- Equivalent to infinite magnetic conductivity (large μ)

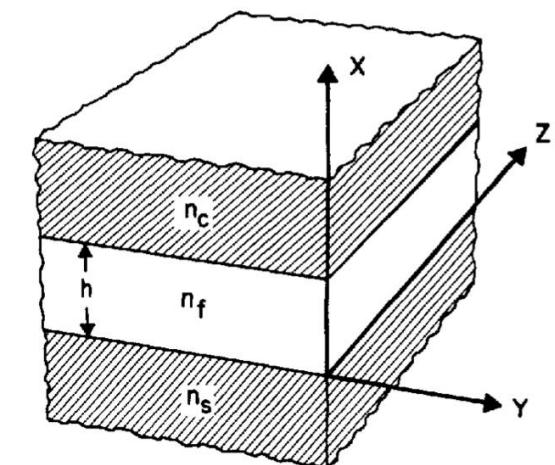
- $B = \mu H_2 \Leftrightarrow H_2 = \frac{B}{\mu} \xrightarrow{\mu \rightarrow \infty} 0$: no electric outside the domain, **the H field is back reflected to the domain, No surface current**



- Examples

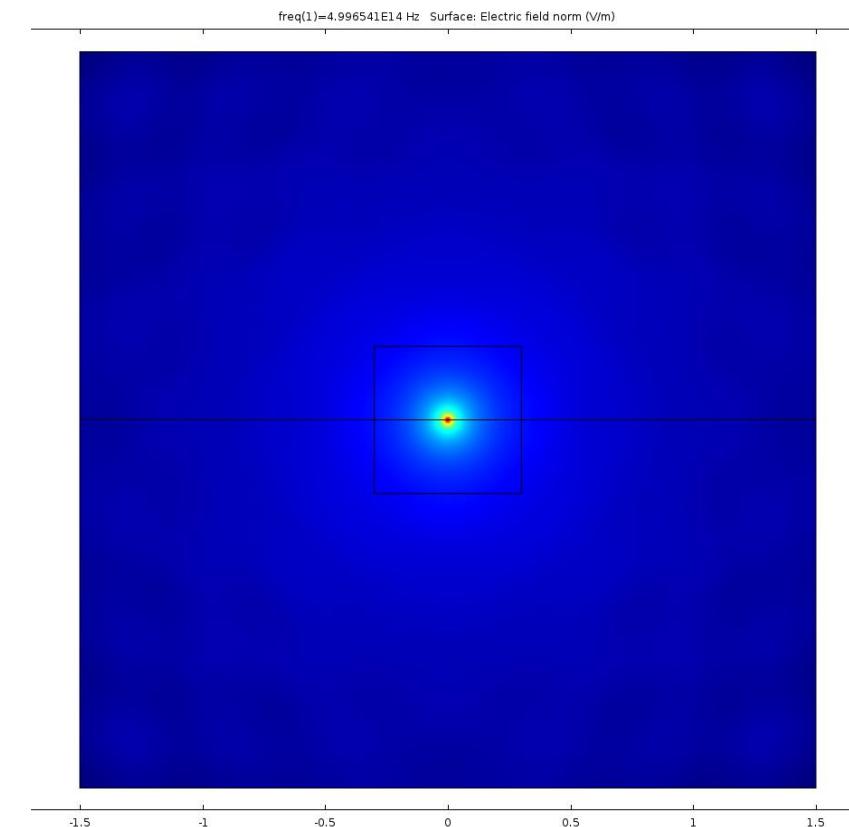
- Interface between dielectric and air
 - Open circuit interface (high impedance)
 - To account for certain symmetry (symmetrical E field, antisymmetrical H field)

$$H_{t,1} = H_{t,2} = 0$$



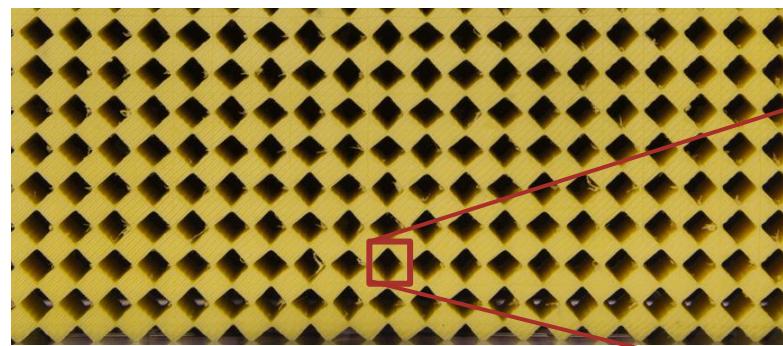
COMSOL: Boundary Conditions

- **Scattering Boundary Condition**
 - Properties
 - Electric field is absorbed → no reflection
 - Examples
 - To simulate an “infinite” domain
 - As we did last time with the point source

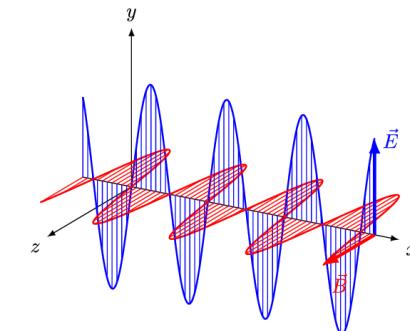
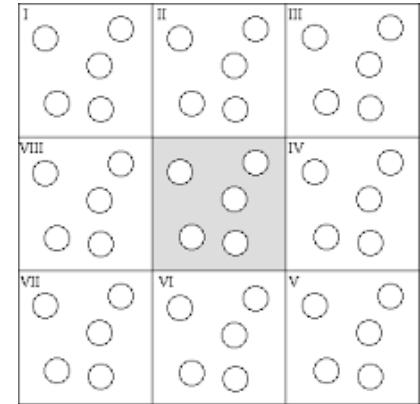
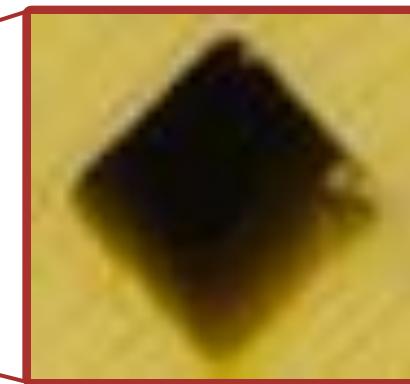


COMSOL: Boundary Conditions

- **Periodic Boundary Condition**
 - For repeating structures
 - Use a unit cell for the analysis
 - Simulates systems expanding infinitely in 1D/2D

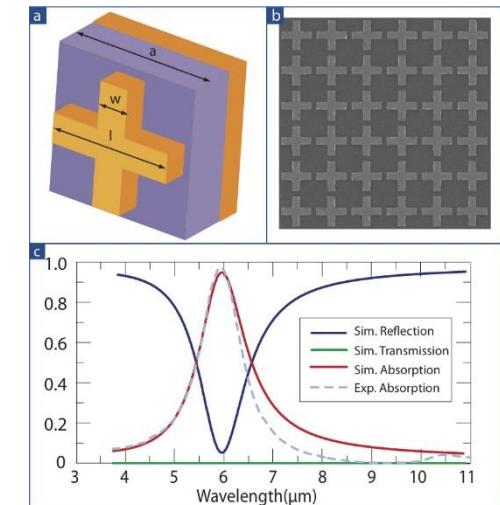
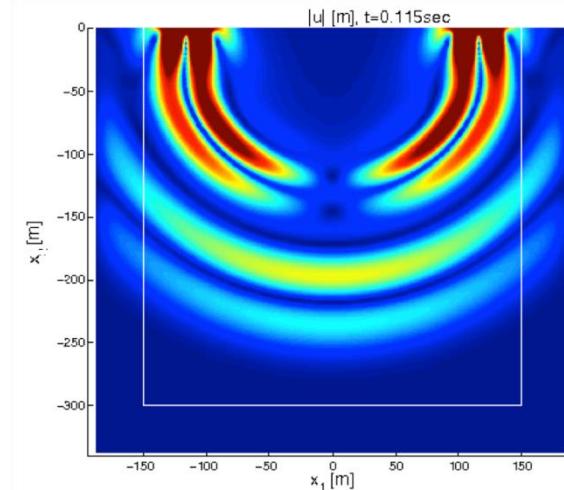


Periodic Boundaries



COMSOL: Boundary Conditions

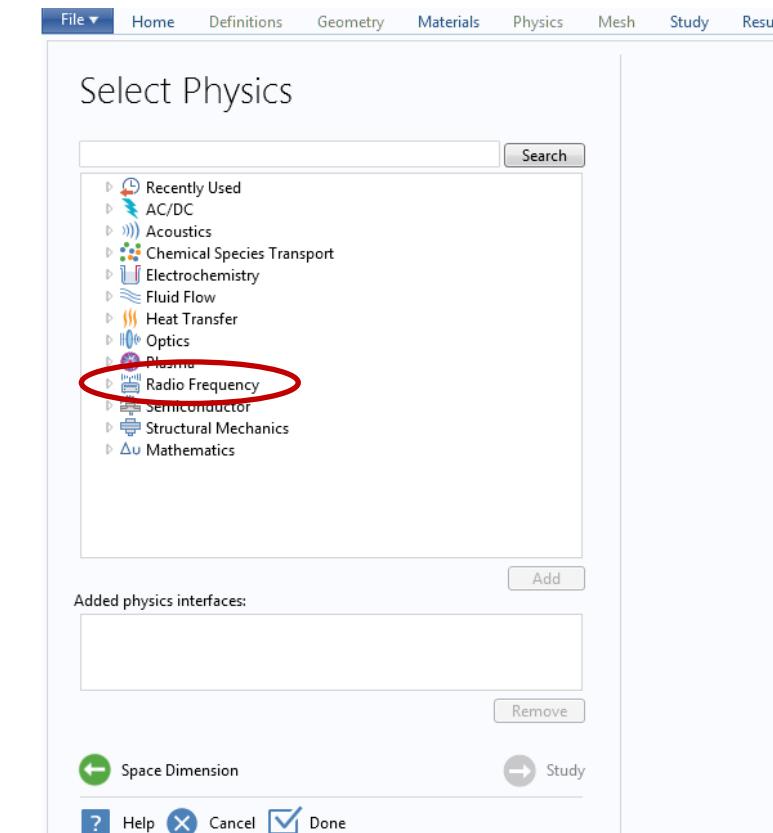
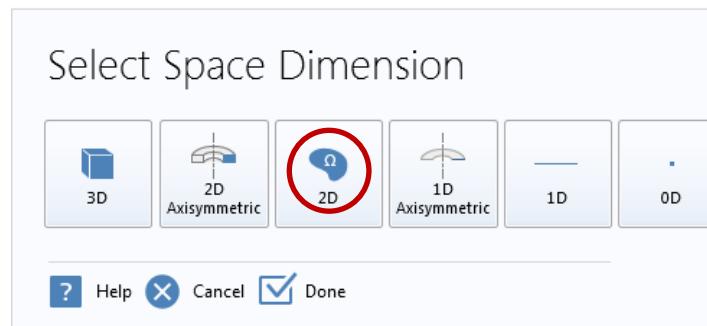
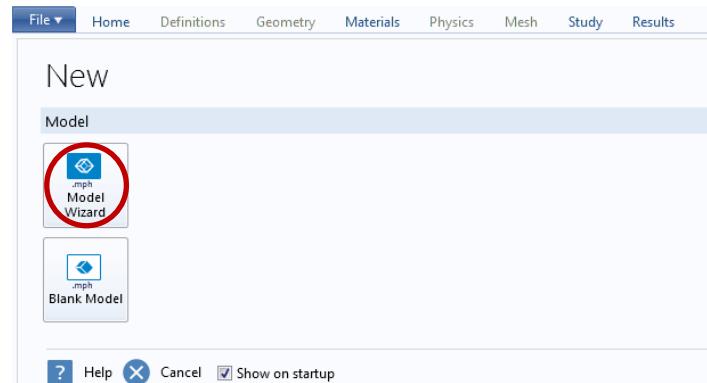
- **Perfectly Matched Layer (PML)**
 - Scattering Boundary
 - Properties
 - Absorbing boundary
 - Truncates EM space in numerical simulations.
 - Possible to simulate open boundary problems.
 - Reduce scattered/reflected waves.
- Examples
 - Antennas
 - Reflectors
 - Absorbing structures
 - Metamaterials



Real life absorber

COMSOL: Last Time

Command to start Comsol: **comsol**



COMSOL: Last Time

File ▾ Home Definitions Geometry Materials Physics Mesh Study Results

Select Physics

The screenshot shows the 'Select Physics' dialog box. On the left, a tree view lists various physics interfaces: Recently Used, AC/DC, Acoustics, Chemical Species Transport, Electrochemistry, Fluid Flow, Heat Transfer, Optics, Plasma, Radio Frequency (which is expanded), Semiconductor, Structural Mechanics, and Mathematics. Under 'Radio Frequency', 'Electromagnetic Waves, Frequency Domain (emw)' is highlighted with a red oval. Below the tree view, there is a list of 'Added physics interfaces' containing 'Electromagnetic Waves, Frequency Domain (emw)'. At the bottom of the dialog are buttons for 'Space Dimension' (with arrows), 'Study' (with arrows), 'Help', 'Cancel', and 'Done' (with a checked checkbox). A 'Search' button is also present at the top right of the dialog.

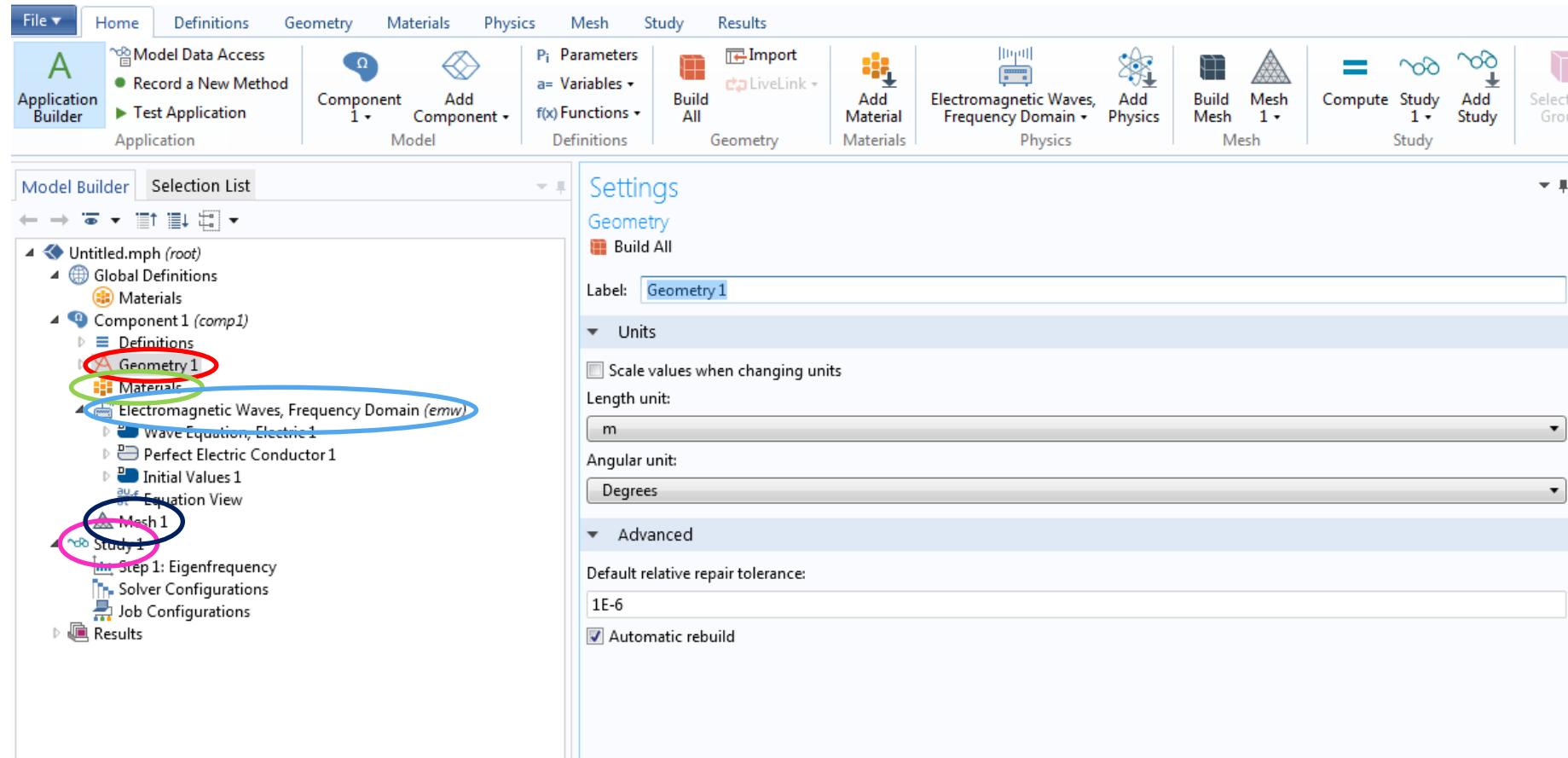
Electromagnetic Waves, Frequency Domain

The Radio Frequency, Electromagnetic Waves, Frequency Domain interface is used to solve for time-harmonic electromagnetic field distributions.

For this physics interface, the maximum mesh element size should be limited to a fraction of the wavelength. The domain size that can be simulated thus scales with the amount of available computer memory and the wavelength. The physics interface supports the study types Frequency Domain, Eigenfrequency, Mode Analysis, and Boundary Mode Analysis. The Frequency Domain study type is used for source driven simulations for a single frequency or a sequence of frequencies. The Eigenfrequency study type is used to find resonance frequencies and their associated eigenmodes in resonant cavities.

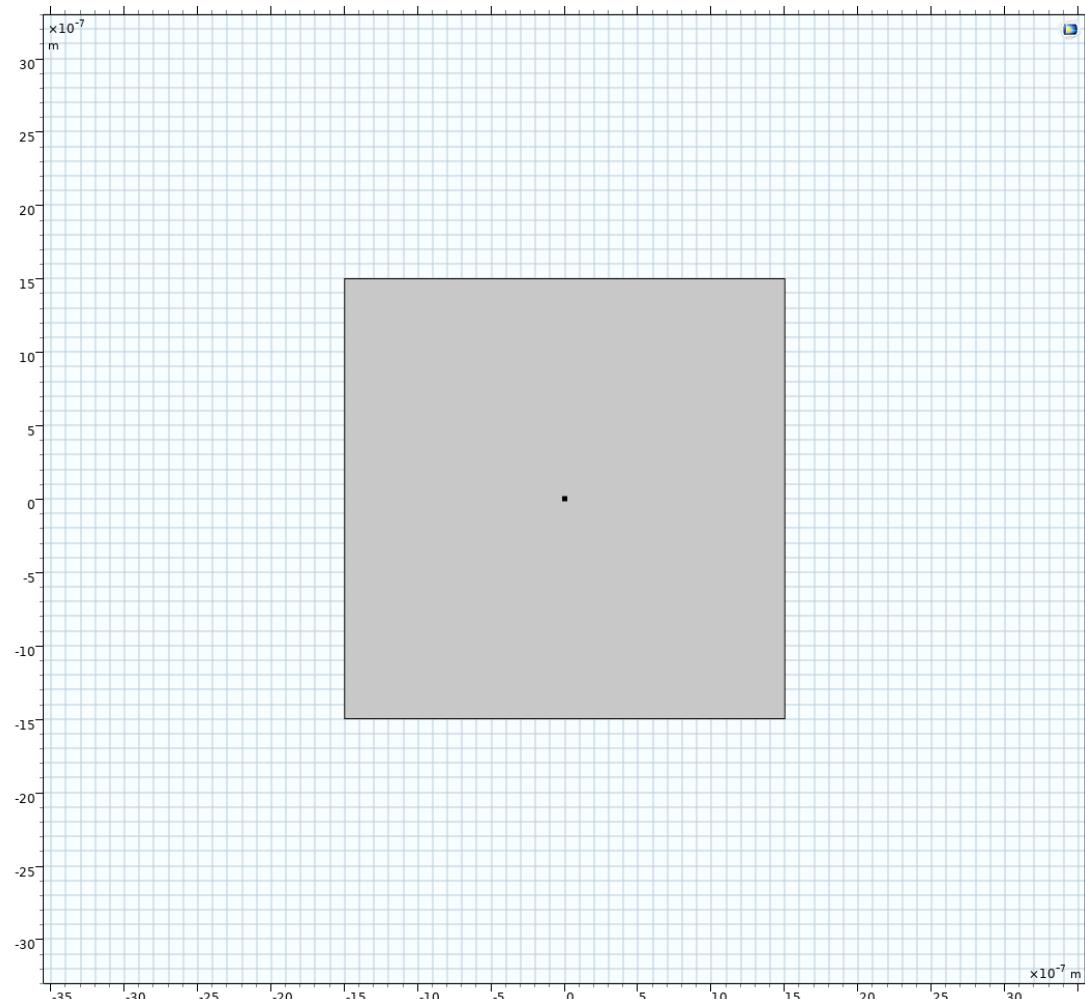
This physics interface solves the time-harmonic wave equation for the electric field.

COMSOL: Last Time



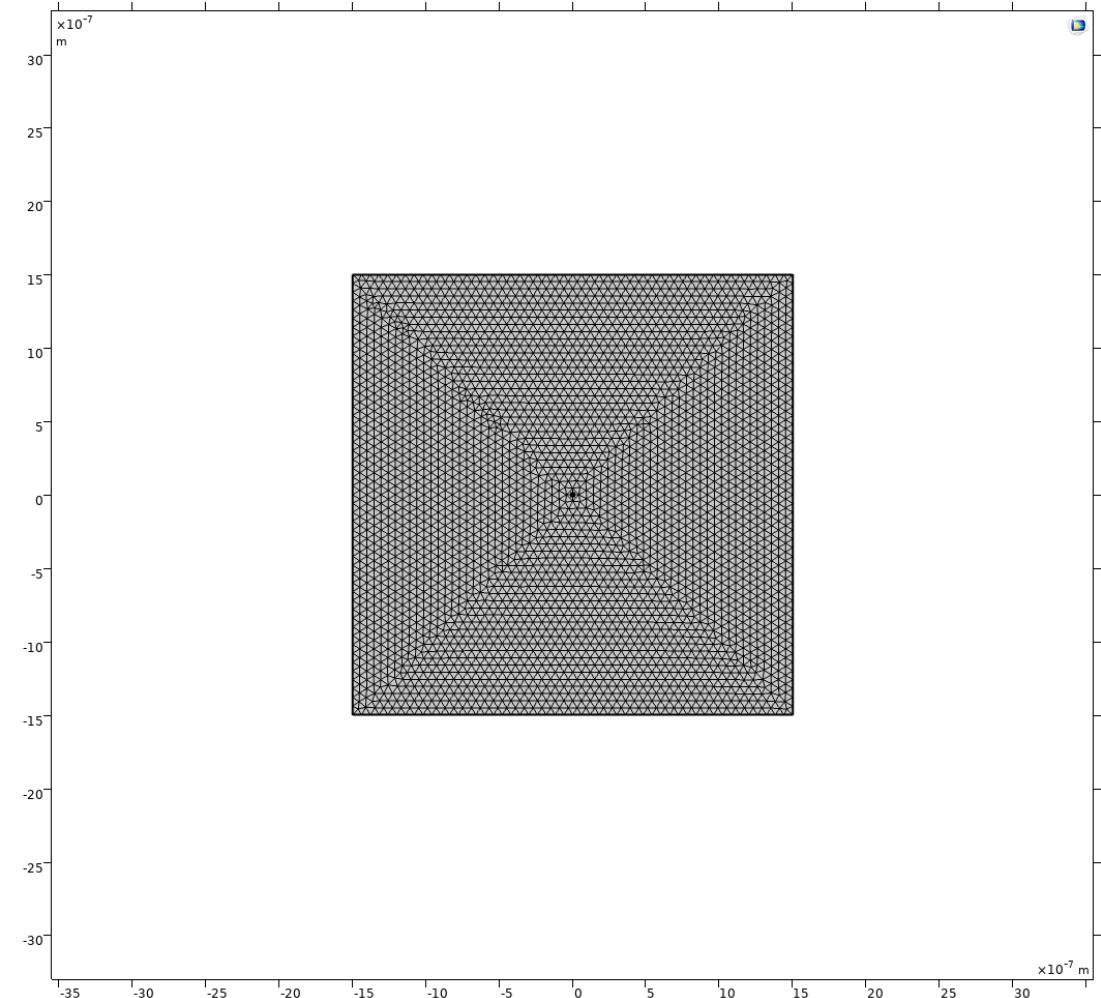
COMSOL: Last Time

- Define simulation domain
- Build geometry

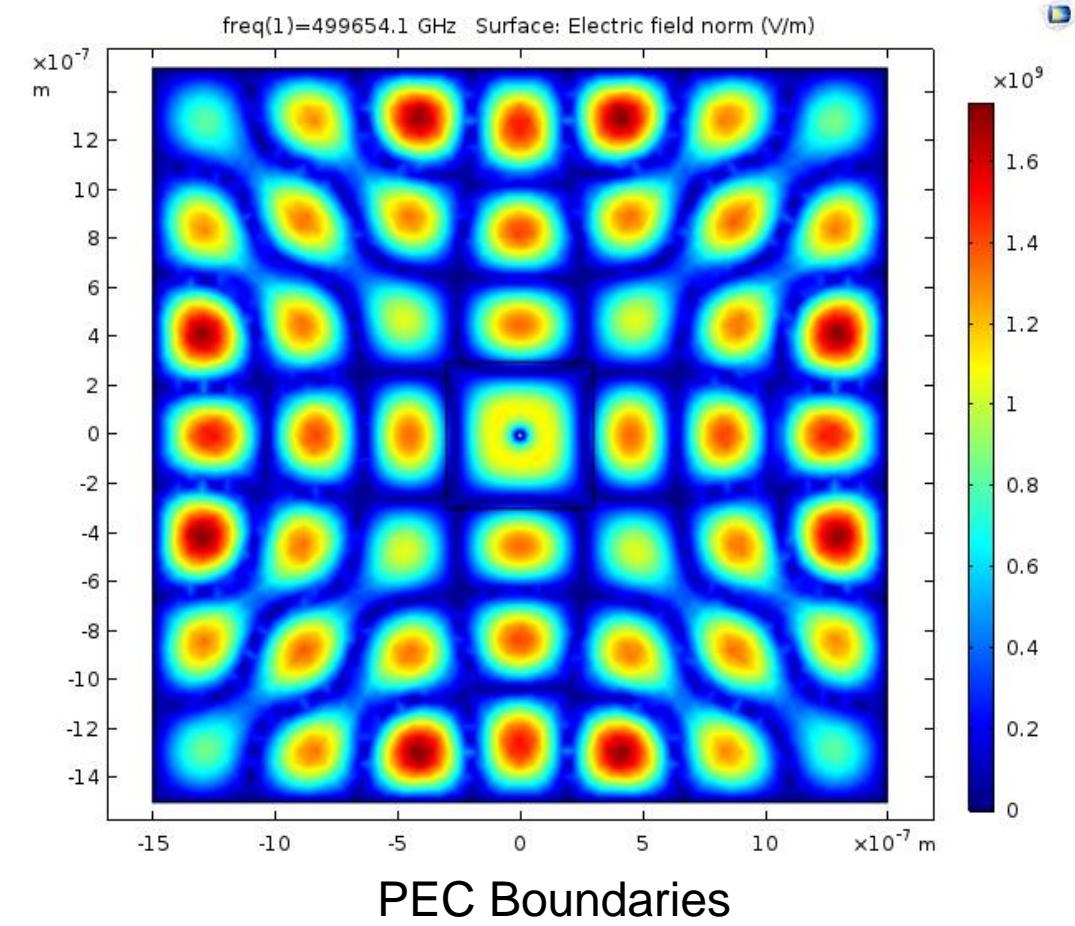
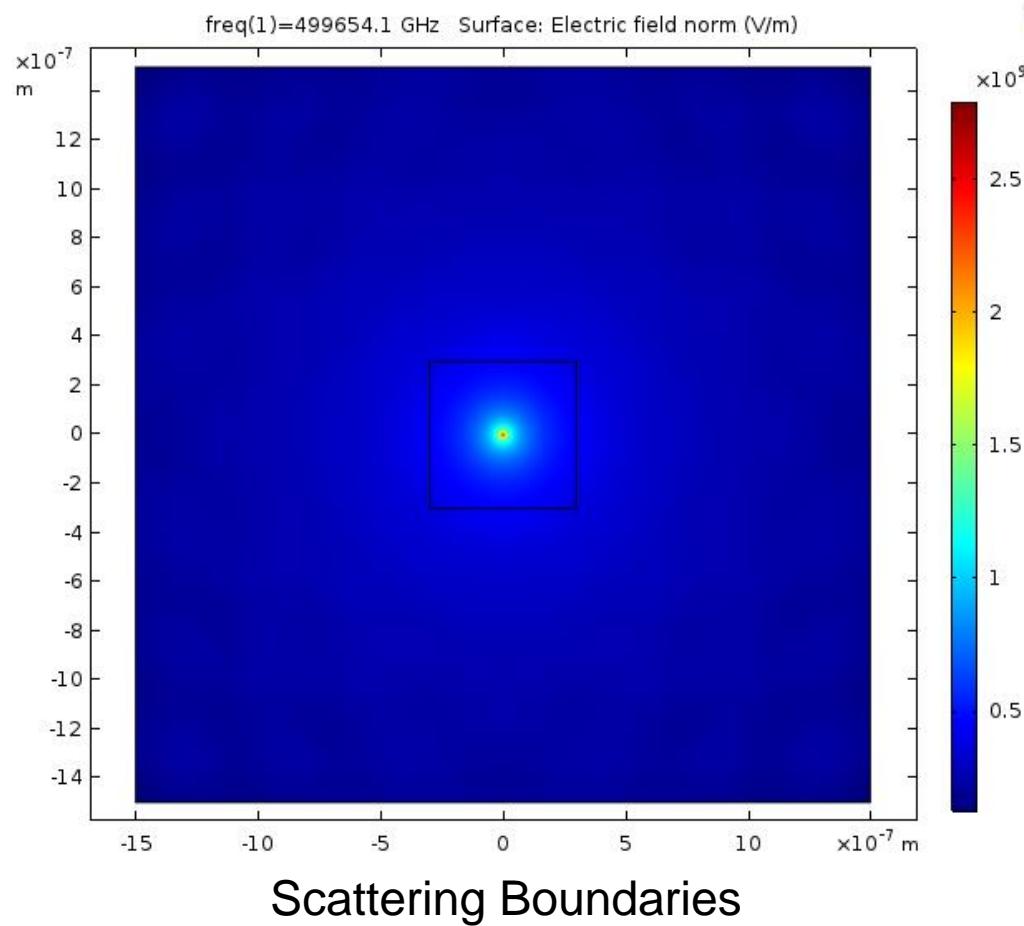


COMSOL: Last Time

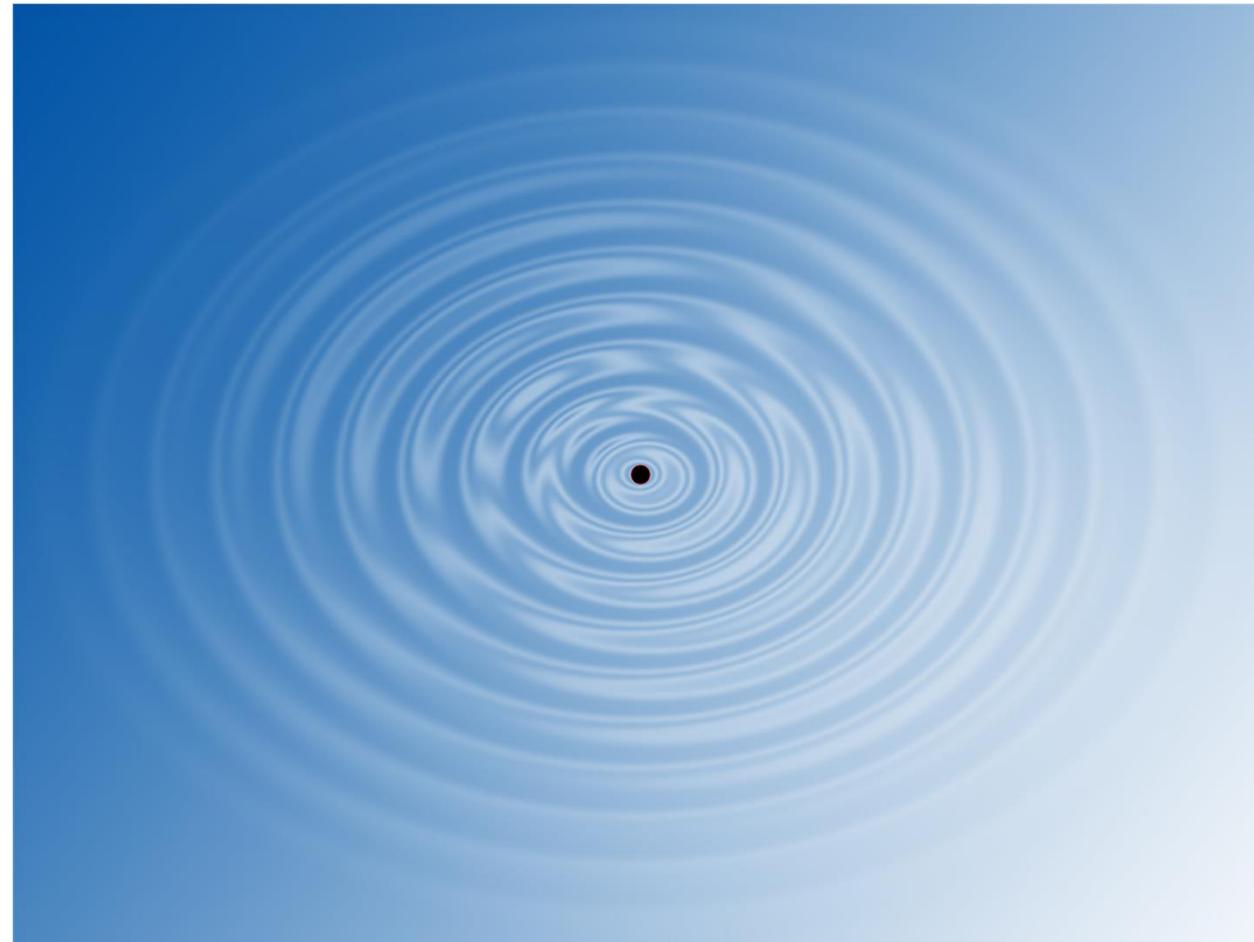
- Discretization of simulation domain
 - Build mesh
- Mesh size determines accuracy of solution
 - Too large mesh \Rightarrow wrong results
- Accuracy vs. simulation time
 - **Today:** Optimization by manual refinement



COMSOL: Last Time

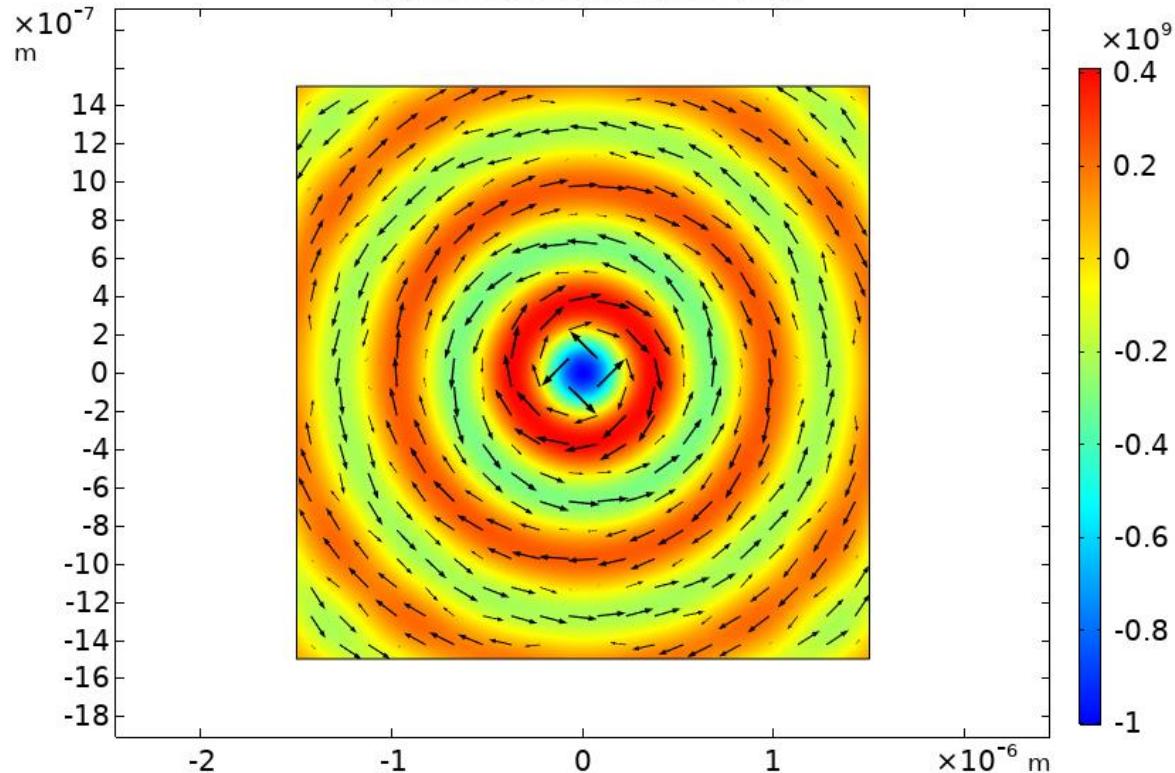


COMSOL: Last Time – Point Source Field



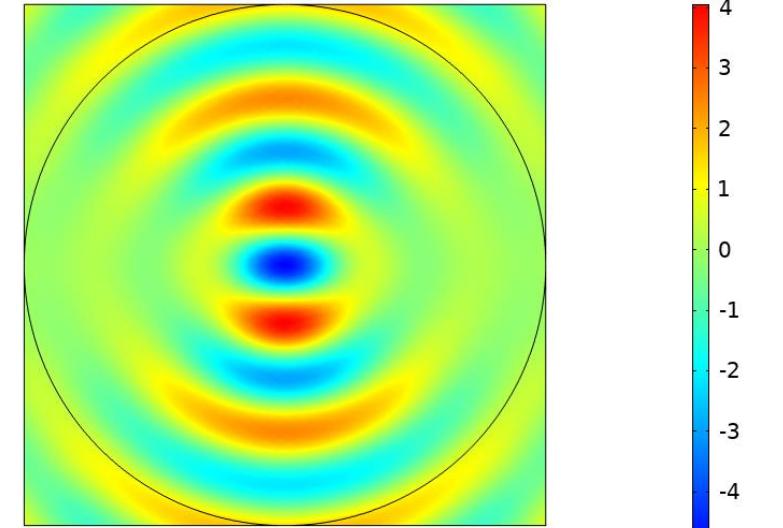
COMSOL: Last Time – Point Source Field

freq(1)=4.9965E5 GHz Surface: Electric field, z-component (V/m)
Arrow Surface: Magnetic field



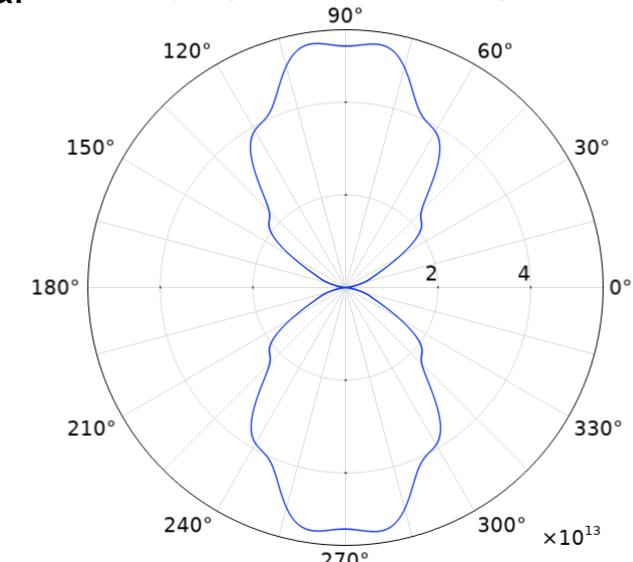
Using line current

freq(1)=4.9965E5 GHz Surface: Electric field, x-component (V/m)

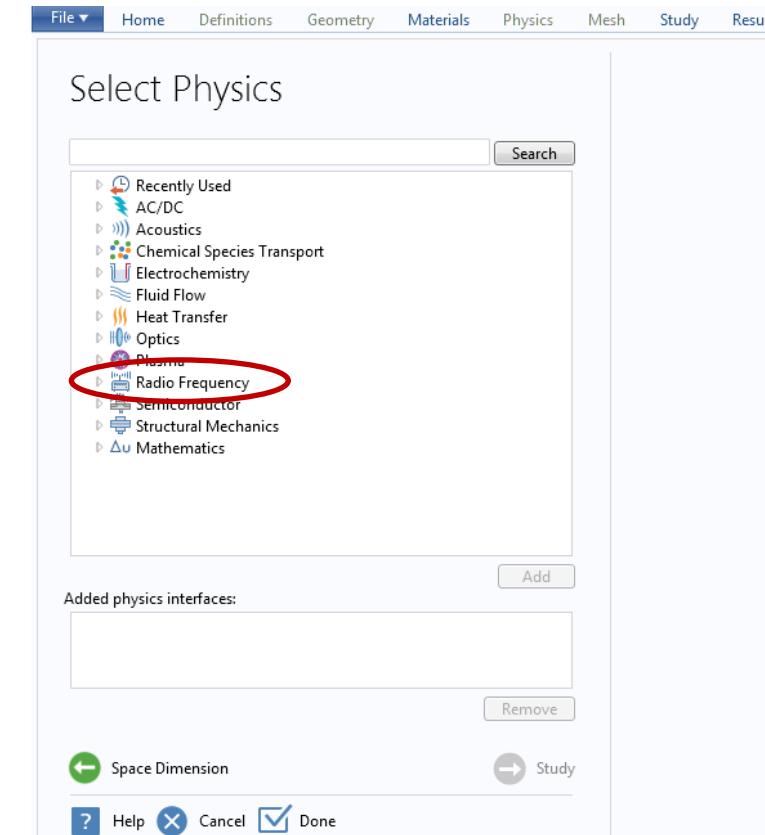
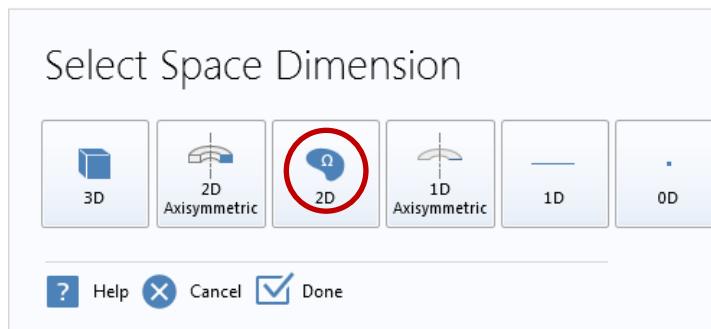
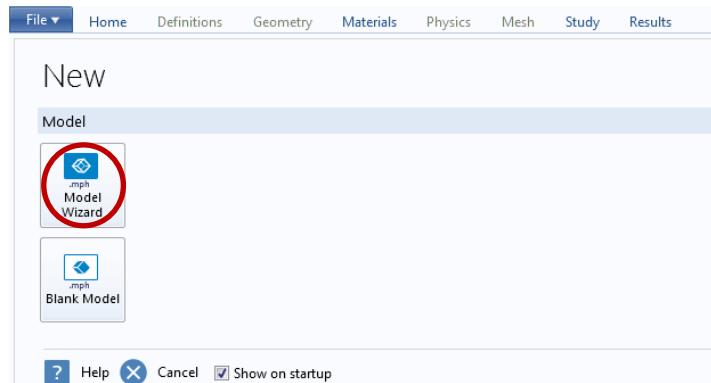


Using electrical
dipole

Line Graph: $\sqrt{(\text{emw.Poavx}^2 + \text{emw.Poavy}^2)}$ (W/m²)



COMSOL: Last Time



COMSOL: Last Time

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Select Physics

The screenshot shows the 'Select Physics' dialog box. On the left, there is a tree view of available physics interfaces. The 'Electromagnetic Waves, Frequency Domain (emw)' node under 'Radio Frequency' is highlighted with a red oval. Below the tree, a list of 'Added physics interfaces' shows 'Electromagnetic Waves, Frequency Domain (emw)'. At the bottom, there are buttons for 'Space Dimension' (with a left arrow), 'Study' (with a right arrow), and a row of buttons for 'Help', 'Cancel', and 'Done' (with a checked checkbox).

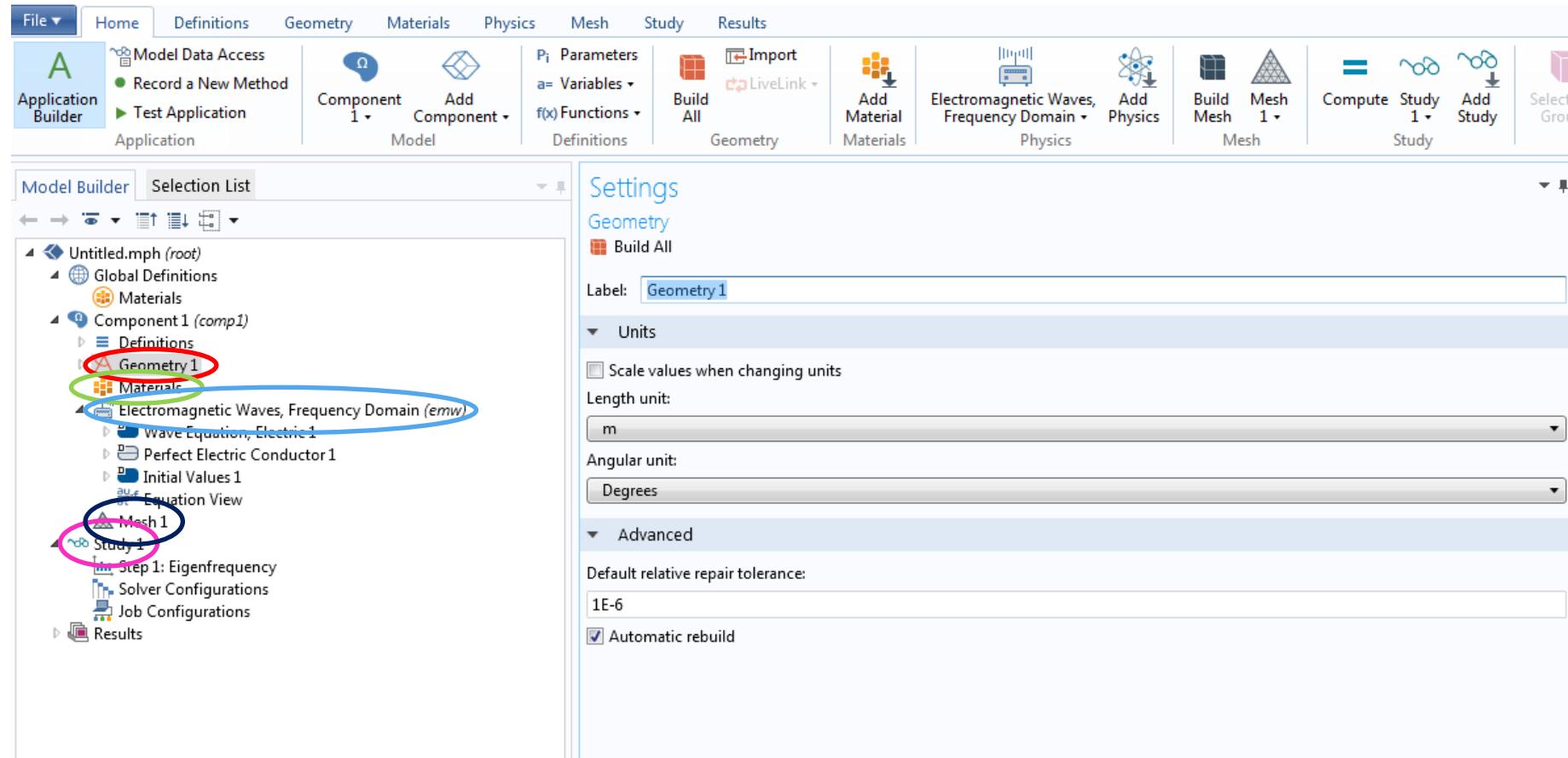
Electromagnetic Waves, Frequency Domain

The Radio Frequency, Electromagnetic Waves, Frequency Domain interface is used to solve for time-harmonic electromagnetic field distributions.

For this physics interface, the maximum mesh element size should be limited to a fraction of the wavelength. The domain size that can be simulated thus scales with the amount of available computer memory and the wavelength. The physics interface supports the study types Frequency Domain, Eigenfrequency, Mode Analysis, and Boundary Mode Analysis. The Frequency Domain study type is used for source driven simulations for a single frequency or a sequence of frequencies. The Eigenfrequency study type is used to find resonance frequencies and their associated eigenmodes in resonant cavities.

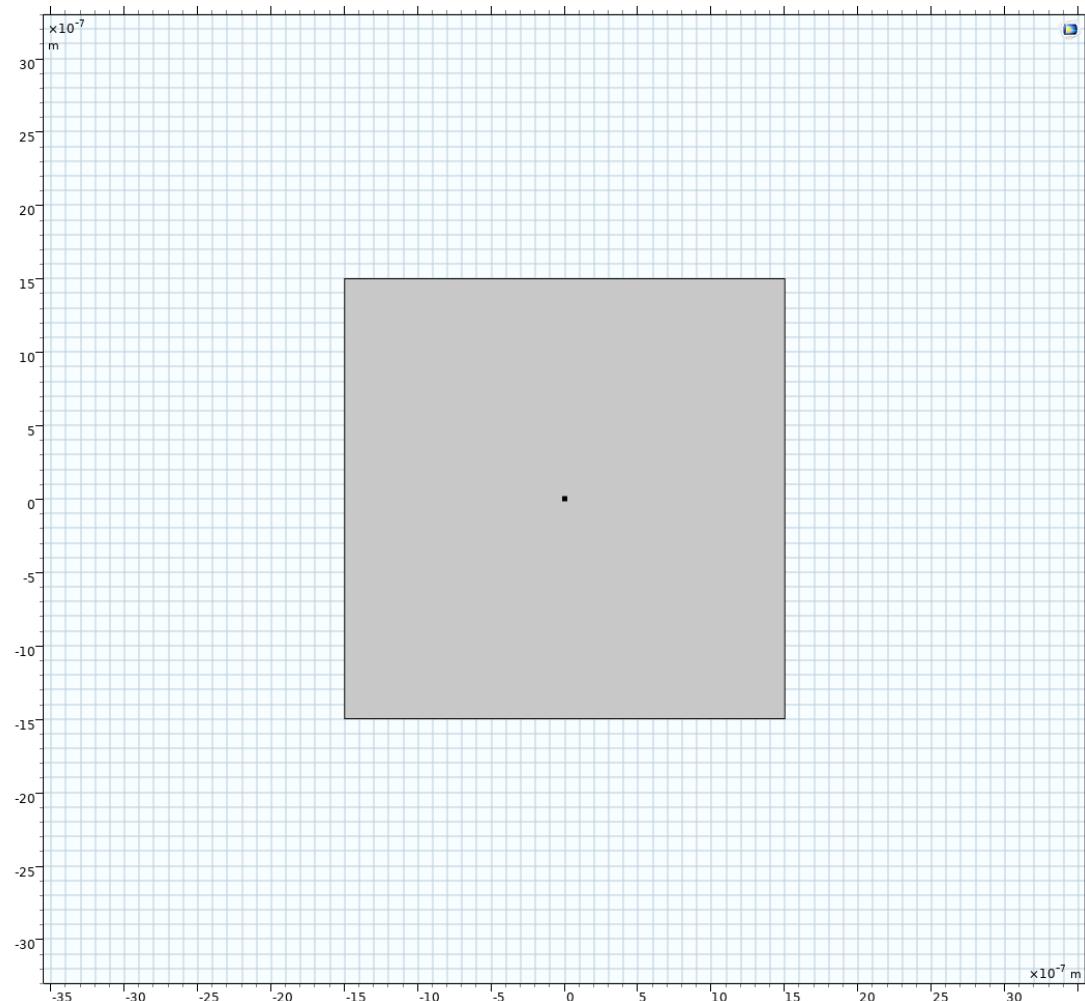
This physics interface solves the time-harmonic wave equation for the electric field.

COMSOL: Last Time



COMSOL: Last Time

- Define simulation domain
- Build geometry

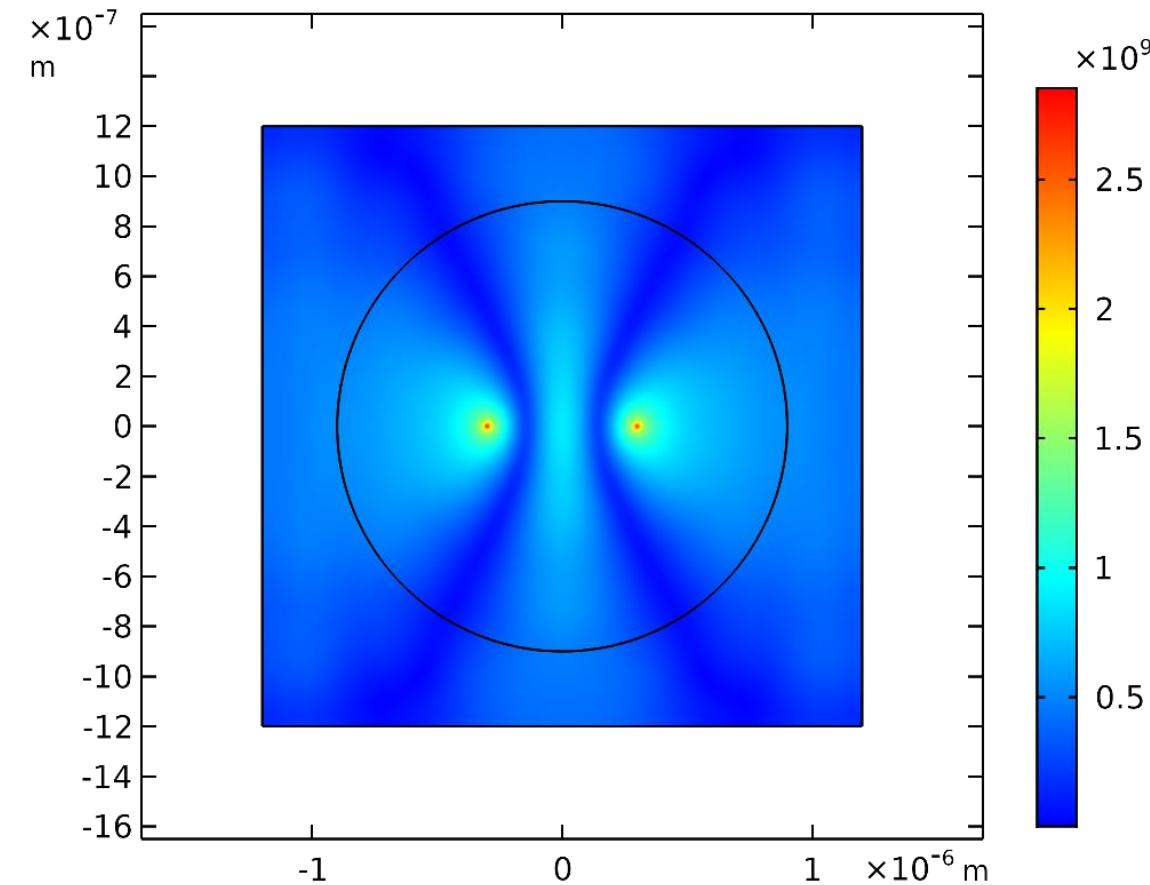


COMSOL: Last Time

- Discretization of simulation domain
 - Build mesh
- Mesh size determines accuracy of solution
 - Too large mesh ⇒ wrong results
- Accuracy vs. simulation time
 - **Today:** Optimization by manual refinement
- Discretization of simulation domain
 - Build mesh
- Mesh size determines accuracy of solution
 - Too large mesh ⇒ wrong results
- Accuracy vs. simulation time
 - **Today:** Optimization by manual refinement

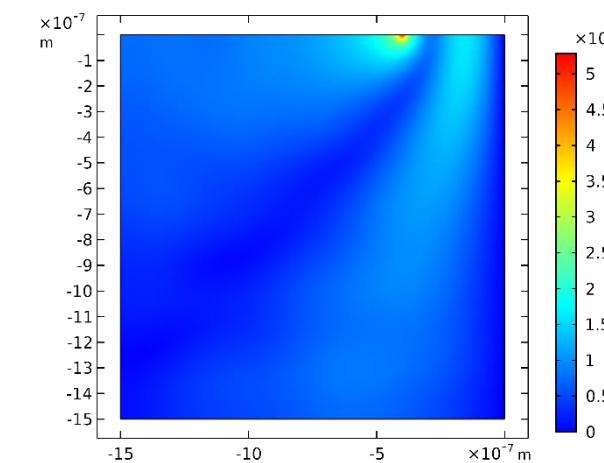
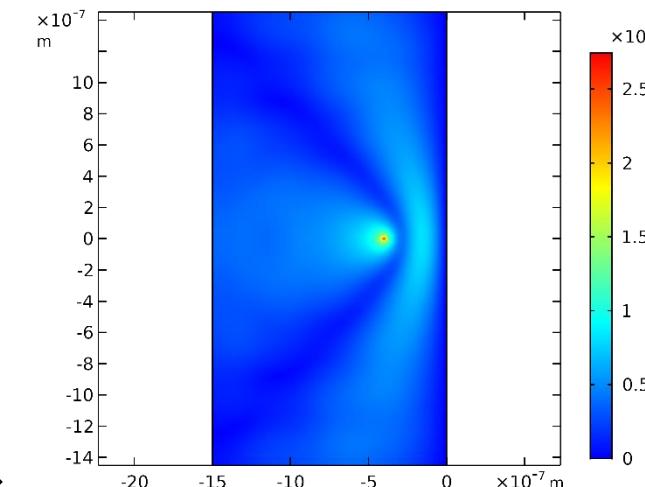
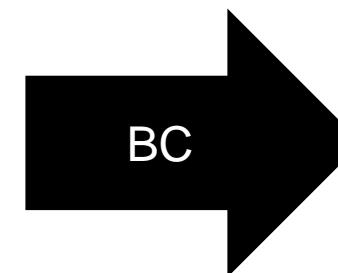
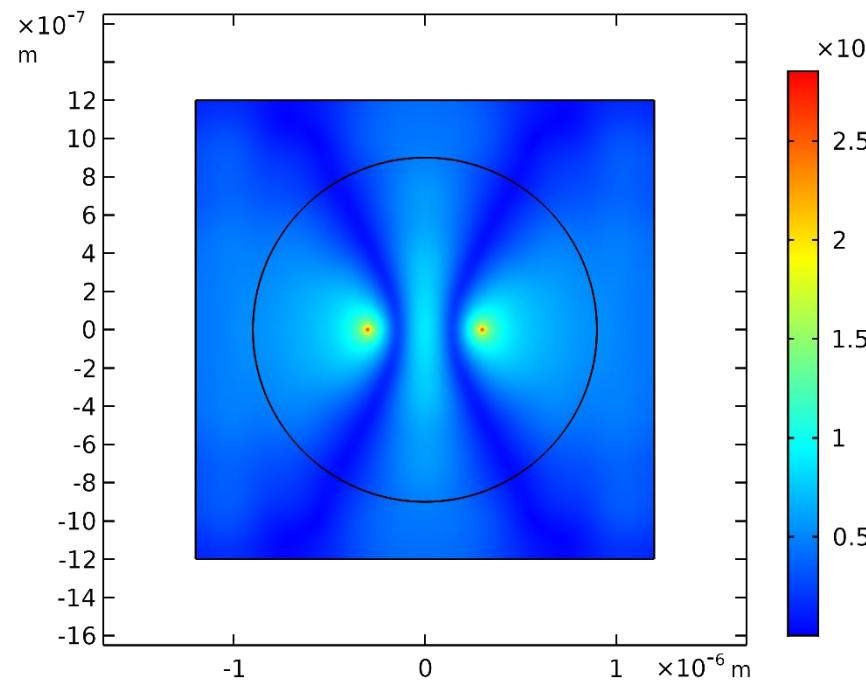
COMSOL: Today

- Mirror effect by using boundary conditions
- Double Source



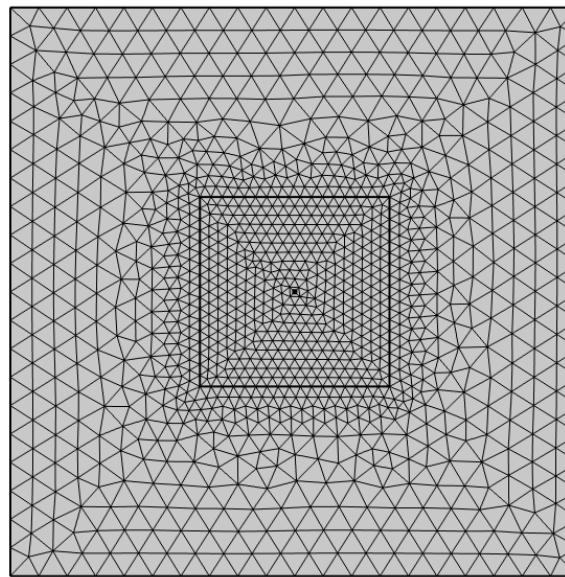
COMSOL: Today

- Mirror effect by using boundary conditions
- Double Source

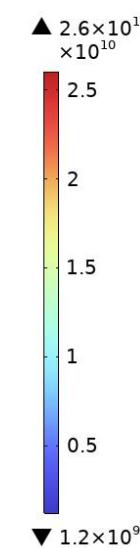


COMSOL: Today

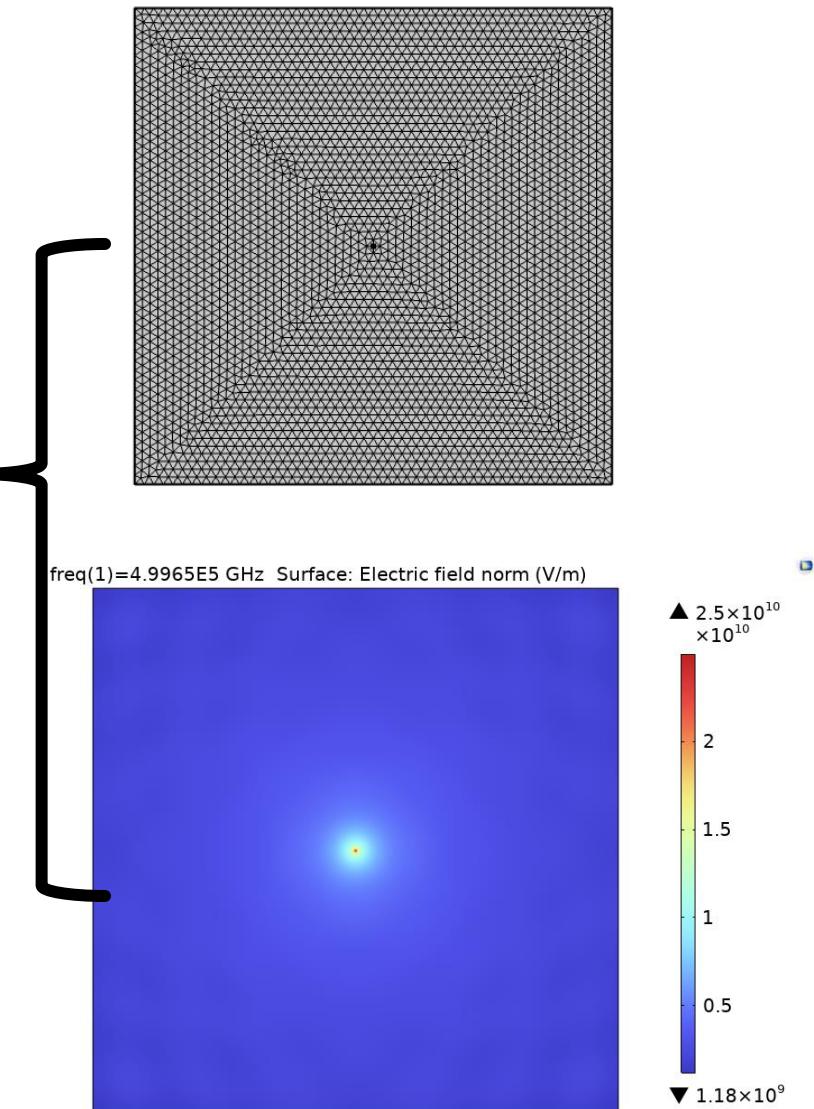
- Accuracy vs computation time
 - Too coarse mesh: incorrect results
 - Too fine mesh: unnecessary large computation time
- Applying manual refinement
 - Refining **only in the region of interest**



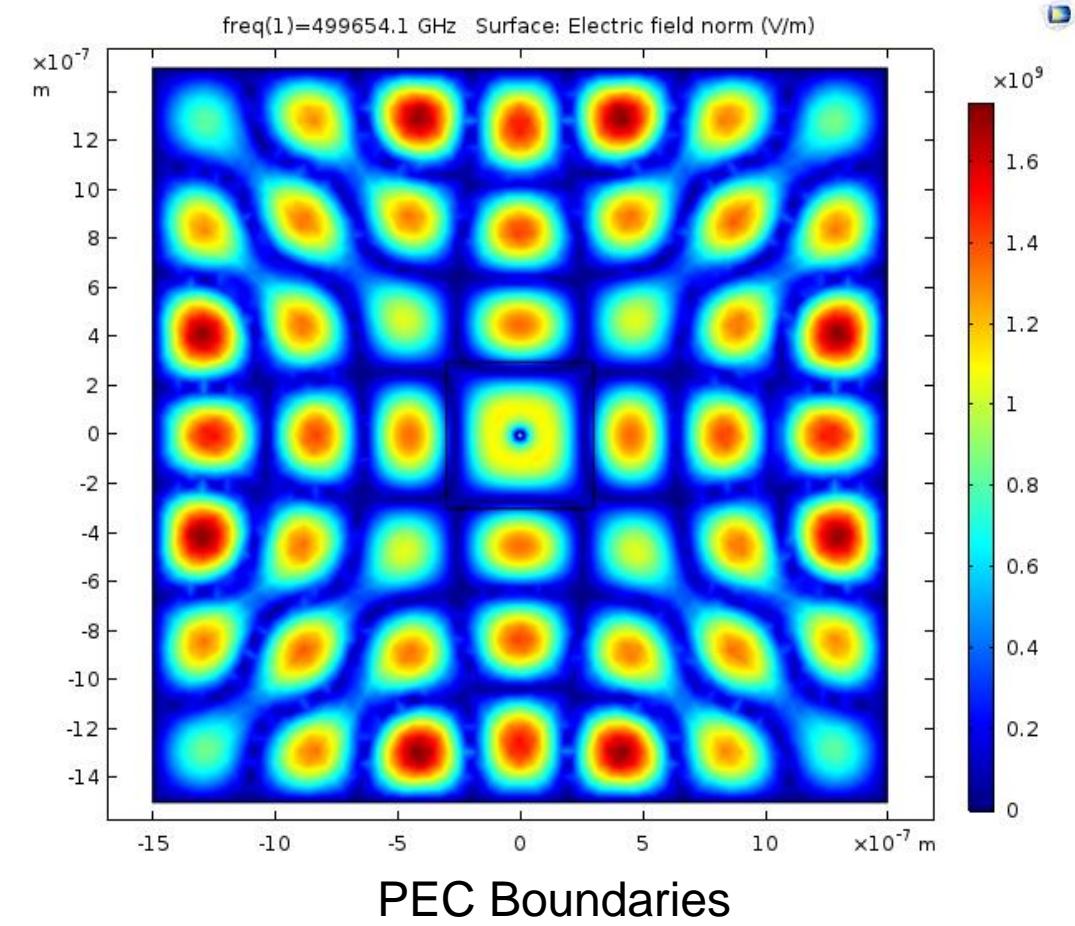
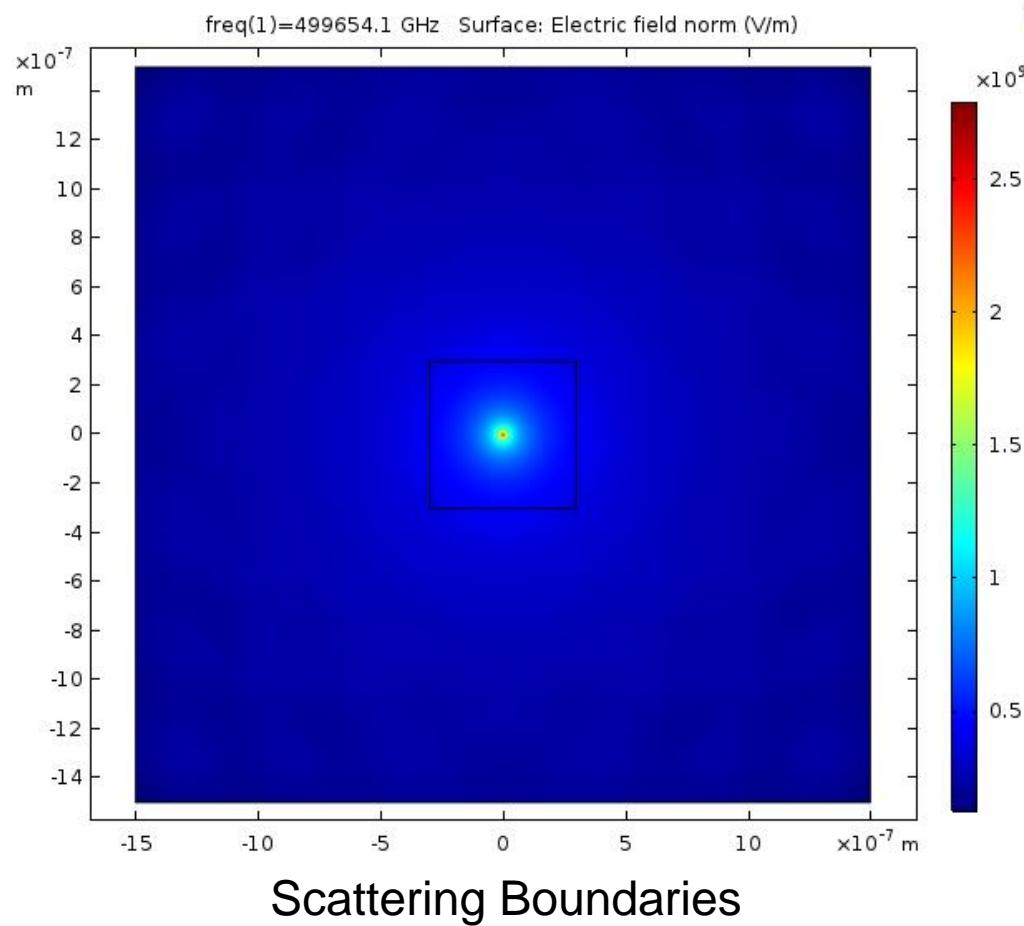
~ 4s



~ 10s

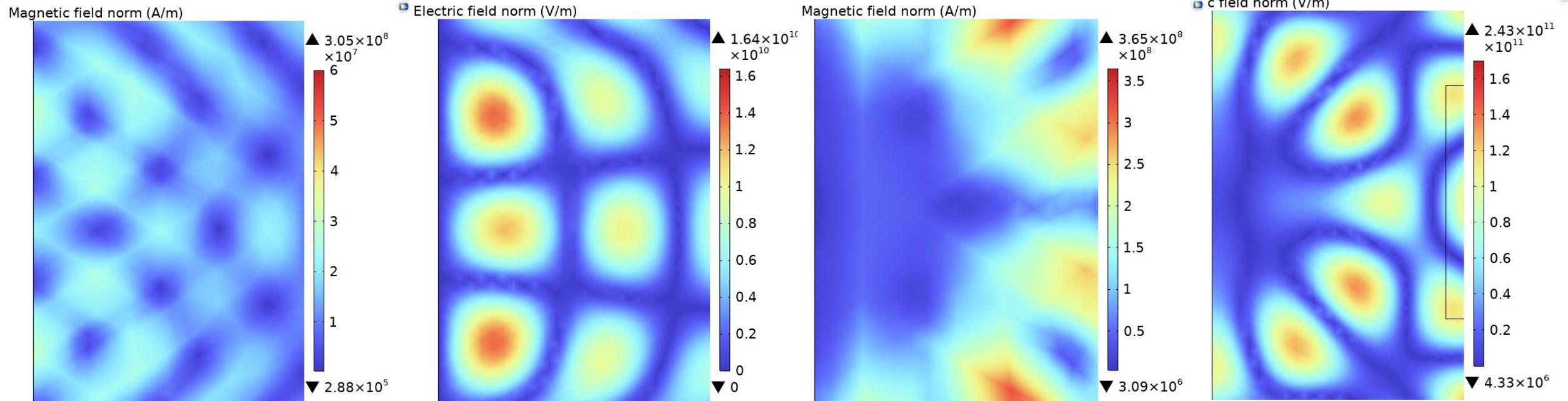


COMSOL: Last Time



COMSOL: Today

- Distinction between PEC and PMC

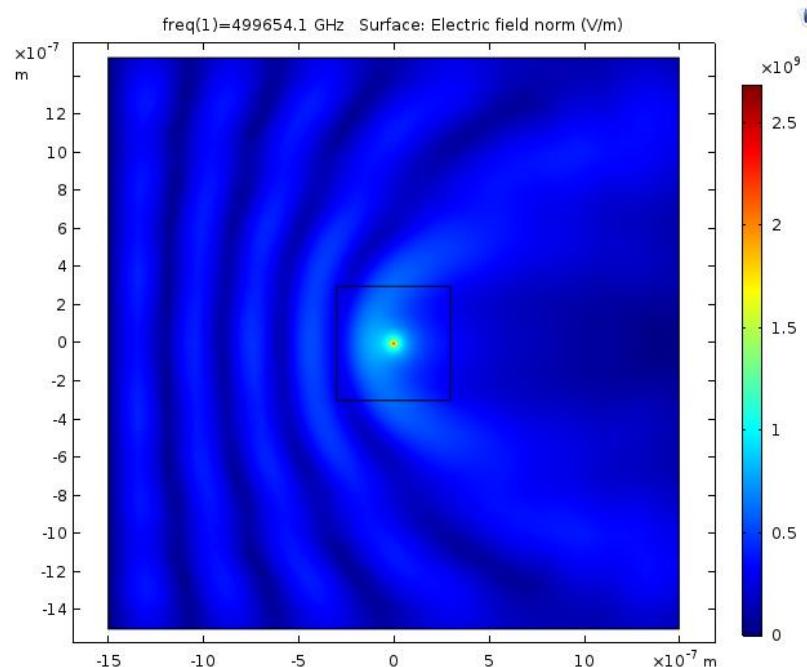


PEC
 $E = 0, H \neq 0$ at boundaries

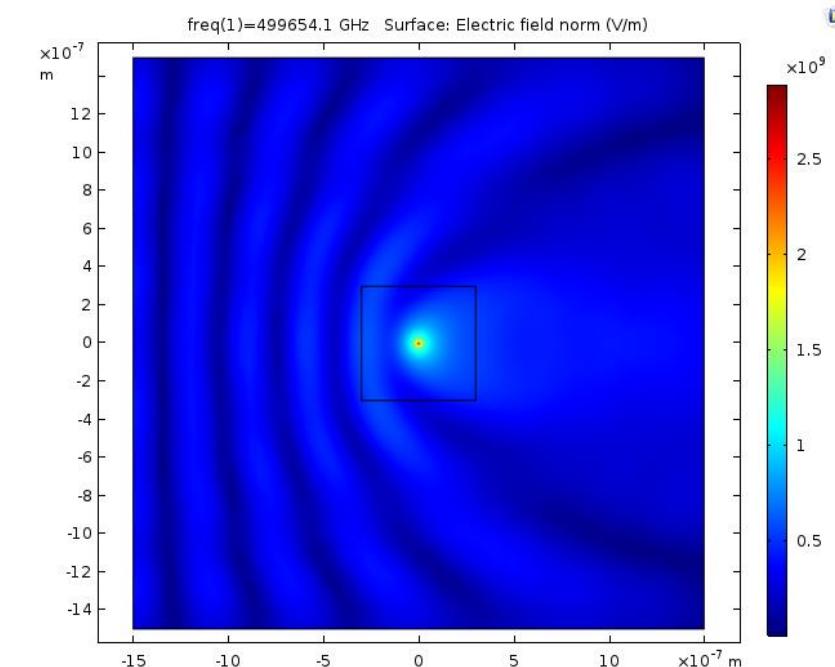
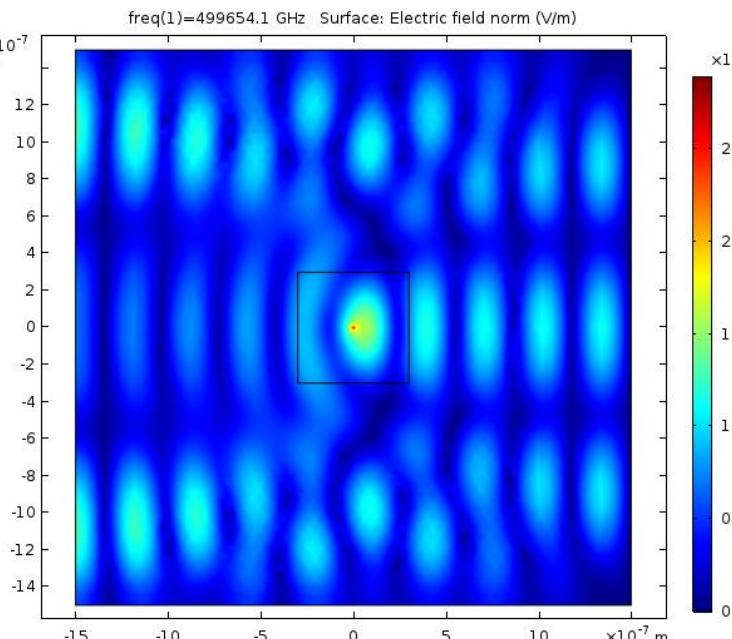
PMC
 $H = 0, E \neq 0$ at boundaries

COMSOL: Today

- Mirror effect by using different boundary conditions

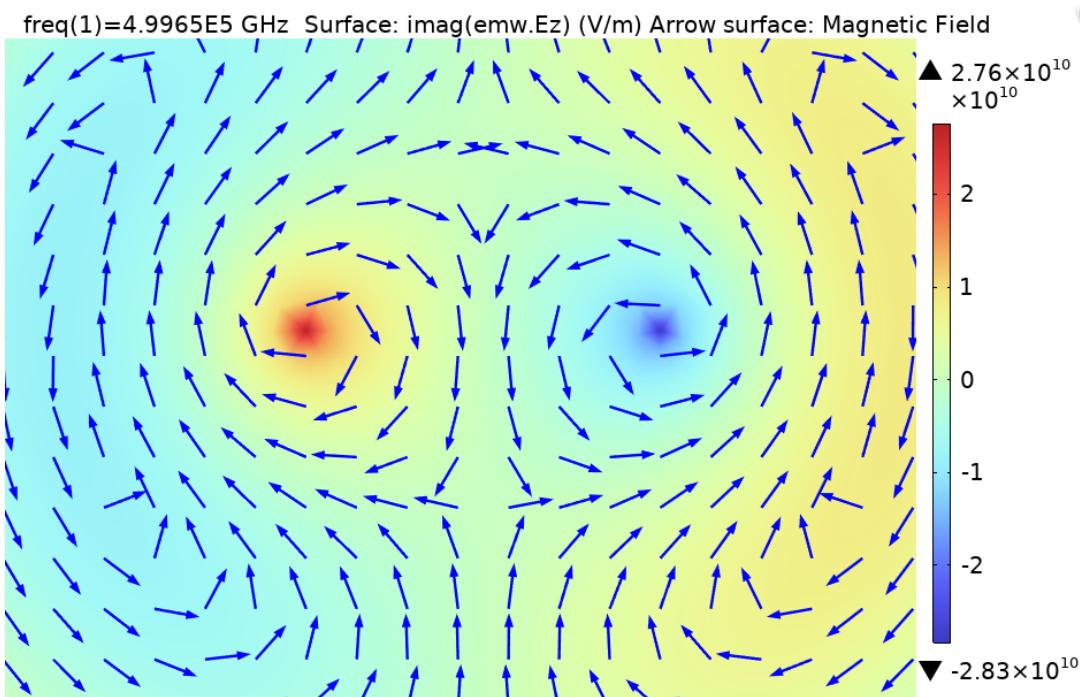


Can you see the difference?
Can you guess the boundary
conditions?

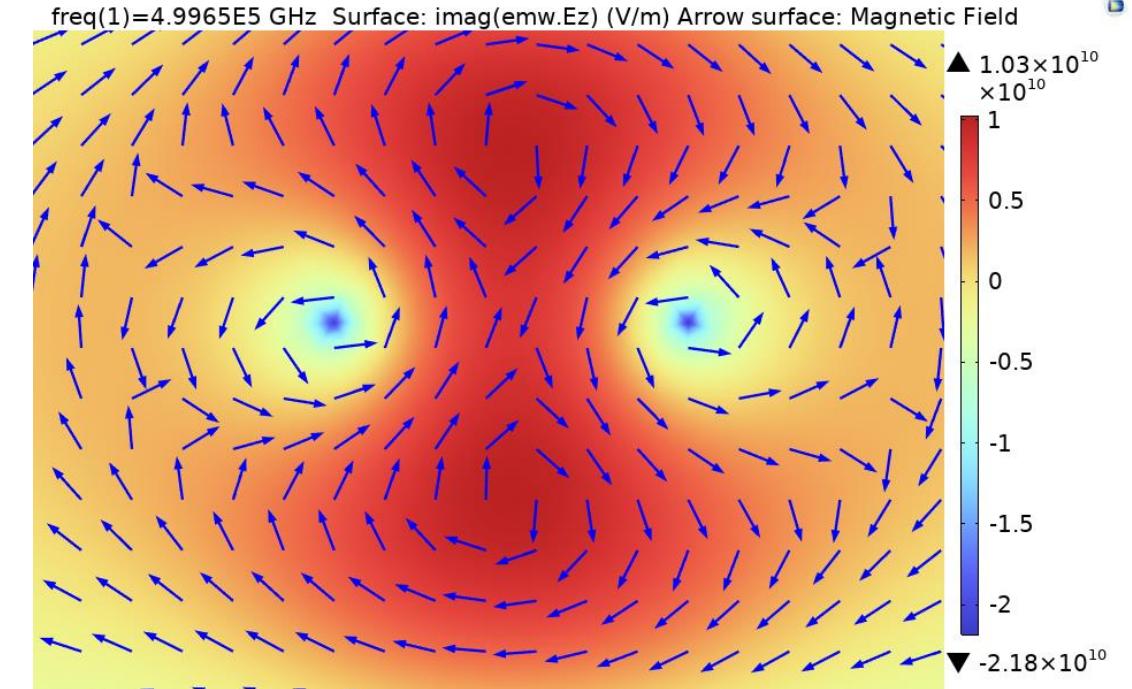


COMSOL: Today

- Double Source



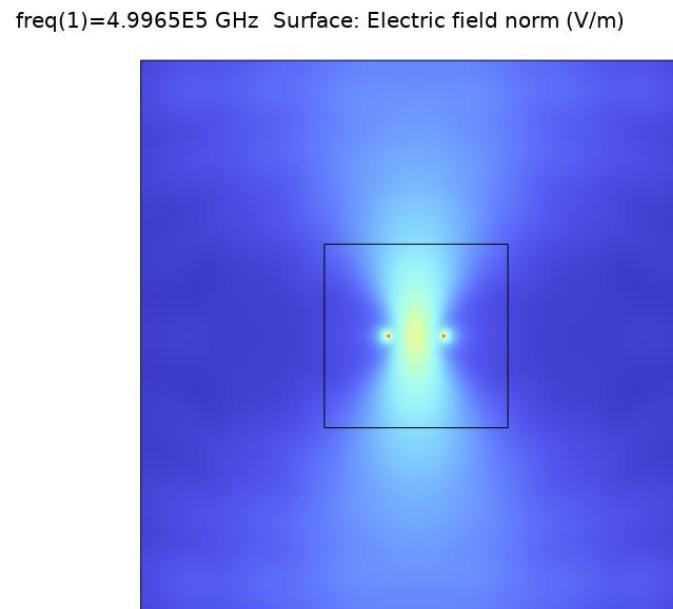
With two line currents in
opposite direction



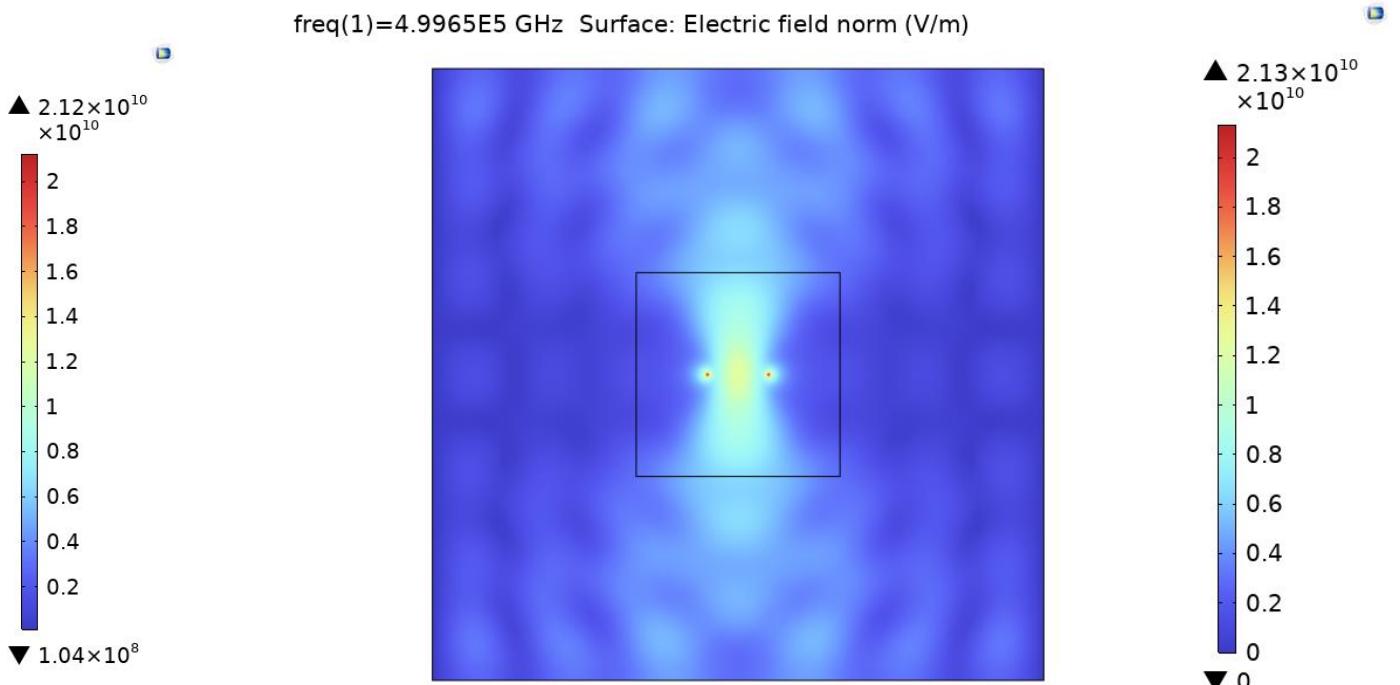
With two line currents in same
direction

COMSOL: Today

- Effects of BCs
 - PECs behave as a mirror interface



SBCs everywhere



PECs at which
boundary?

COMSOL: Today

- Optimizing a parameter → parameter sweep
 - It allows us to evaluate our model's properties w.r.t this parameter

