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P&S COMSOL® Design: Simulations of Optical Components Lecture 4: Wave Optics and Waveguiding

Manuel Kohli & Raphael Schwanninger

Content

- Last week
  - Young’s single/double slit experiment

- Today
  Review on material properties
  - Waveguide
    - Motivation
    - Theory
    - COMSOL

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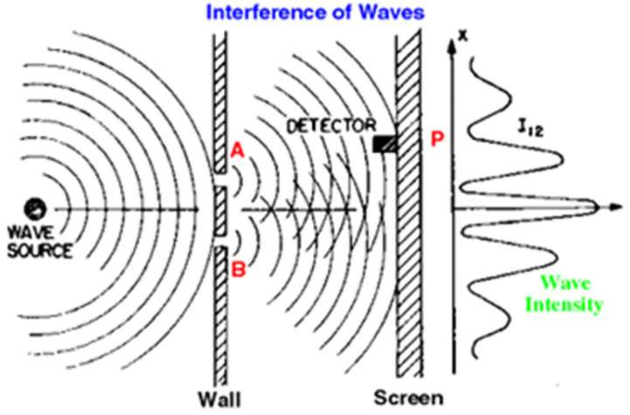
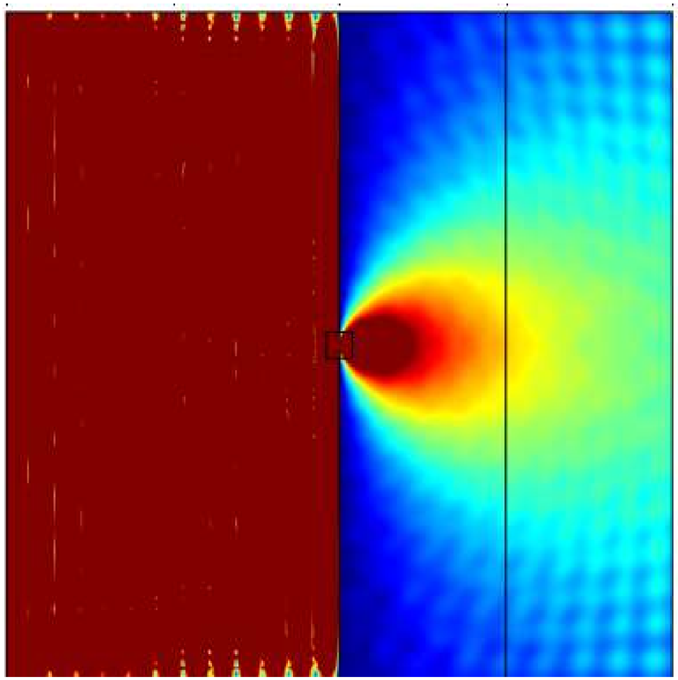
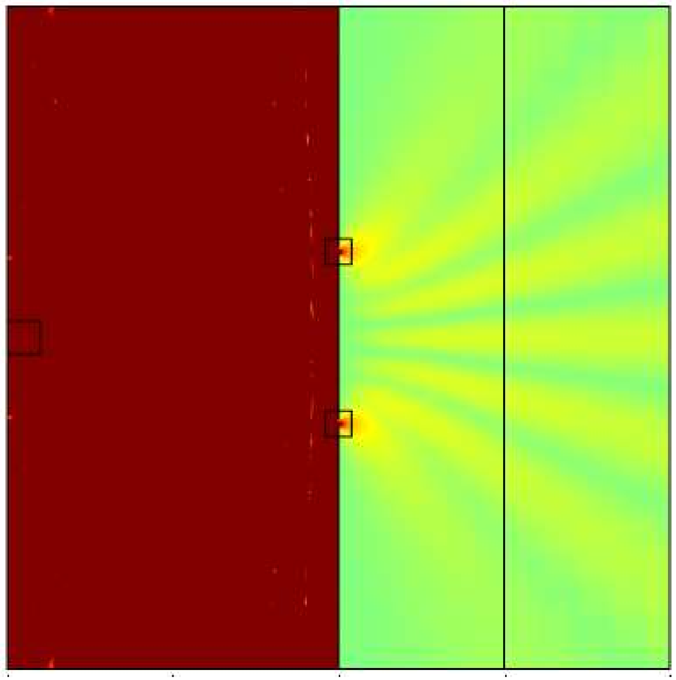
Last Week: Young's Slit Experiments



One Slit



Two Slits



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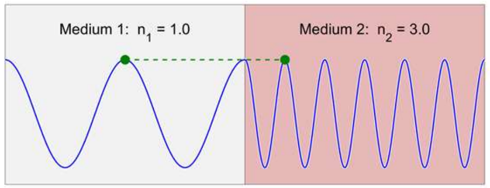
Review: Material Relations

- In order to analyze an EM problem we need to define the **material properties** involved
- Materials are defined by their **refractive index $\boldsymbol{n}$** which is defined as
  - $\boldsymbol{n} = \sqrt{\boldsymbol{\mu_r \varepsilon_r}}$, for vacuum $n = 1$
  - n is a complex number $n = n' + ik$

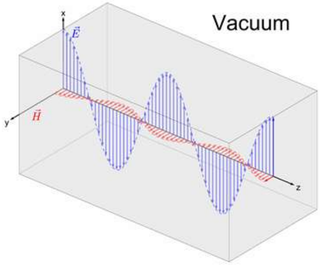
Influences wavelength (n') — Influences losses (k)

Antenna

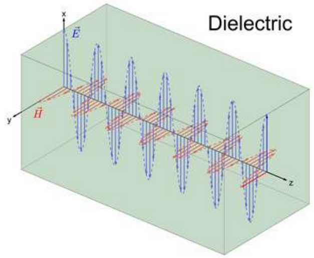
Air — Glass



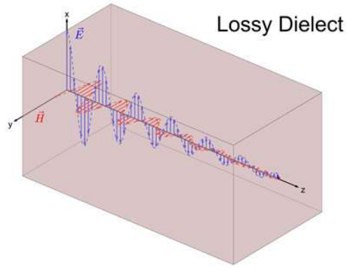
$n = 1$



$n = 3$



$n = 3 + 2i$



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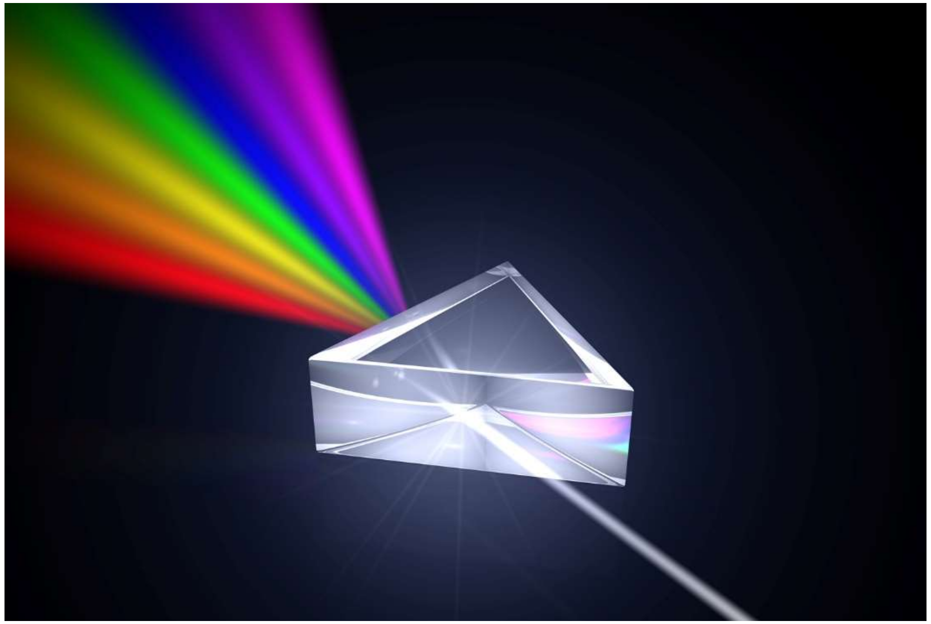
Material Properties

- What is the meaning of $n(\omega)$?
- Newton discovered that it changes with wavelength!



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Waveguiding: Motivation

- How does the internet work?

- Data transfer
  - Wireless
  - By cable

Electrical (copper) or optical (glass fiber)

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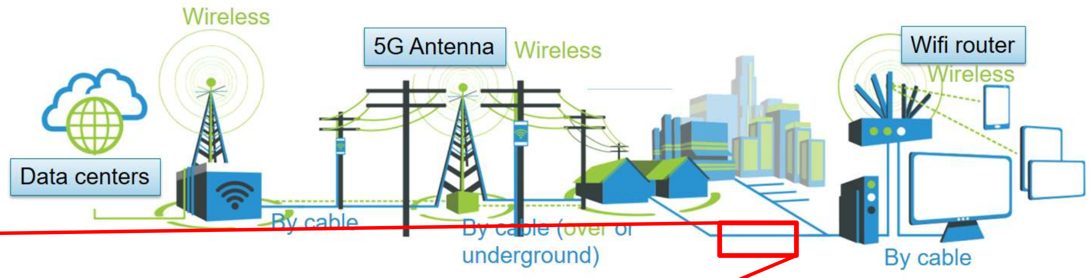
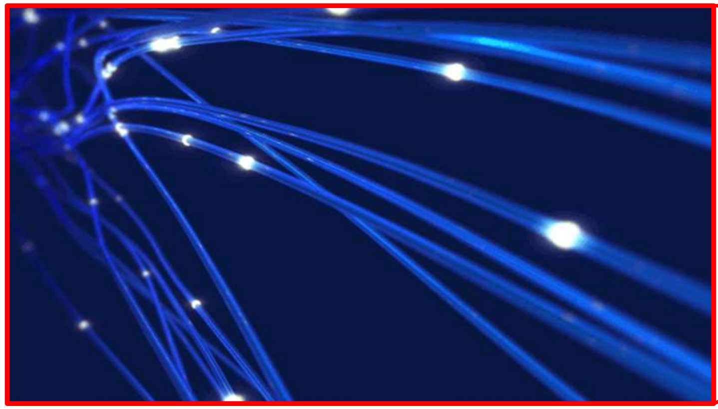
Waveguiding: Motivation

- Data transfer with an optical fiber

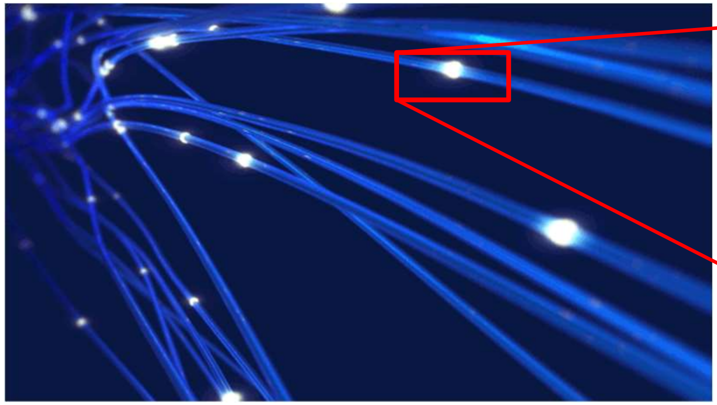


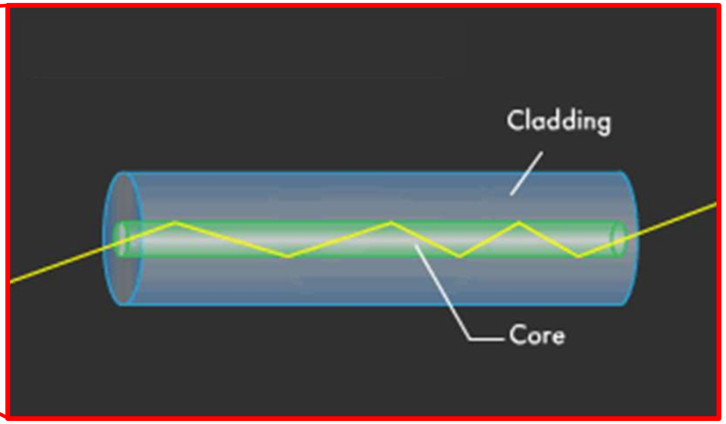


- Use the propagating light in the fiber to transmit information

Waveguiding

- Optical data transmission → in the future maybe also in electronic devices (smartphones etc.)
- Our goal: learn how to simulate optical components used on-chip





How can we make this happen?

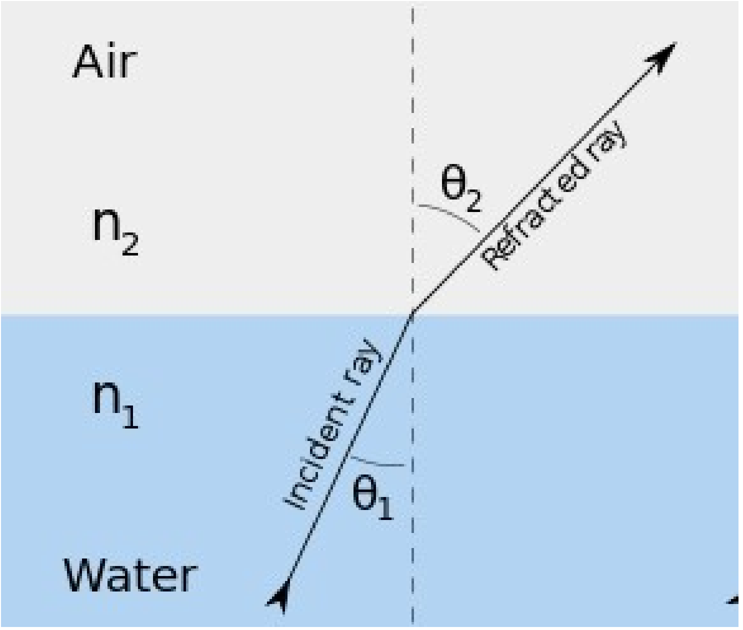
- Optical fiber often used terms:
  - Core
  - Cladding

Waveguiding: Theory

- What happens at the boundary between two materials?

Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

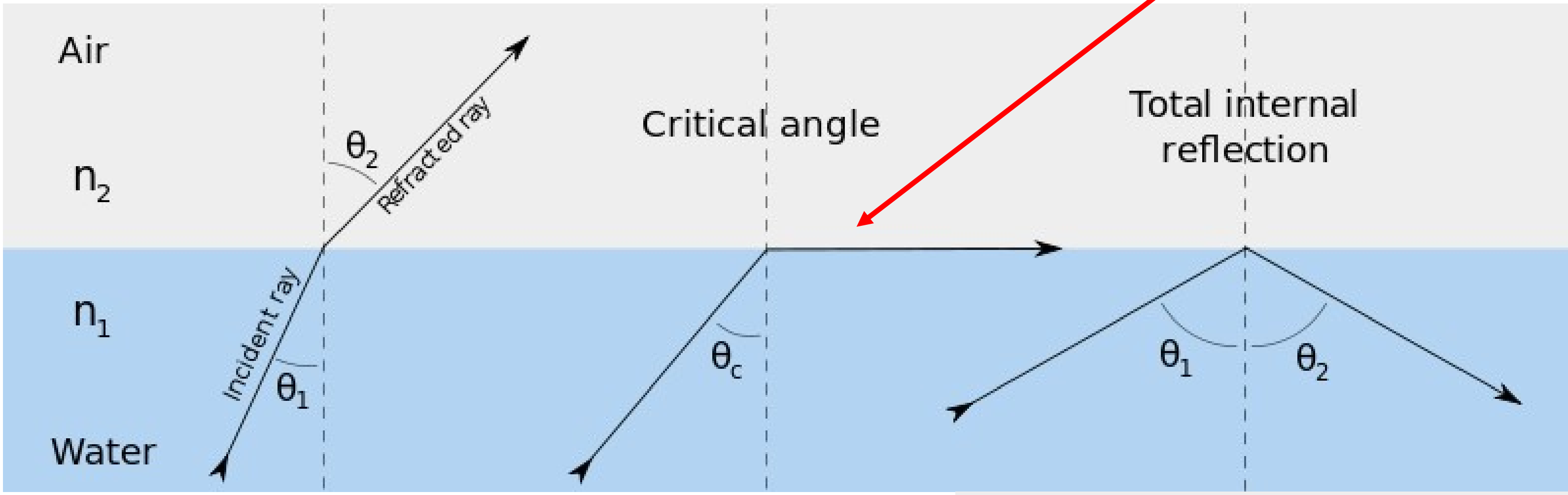


Waveguiding: Theory

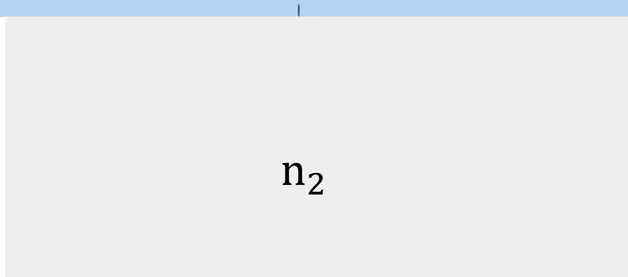
Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- What happens at the boundary between two materials?

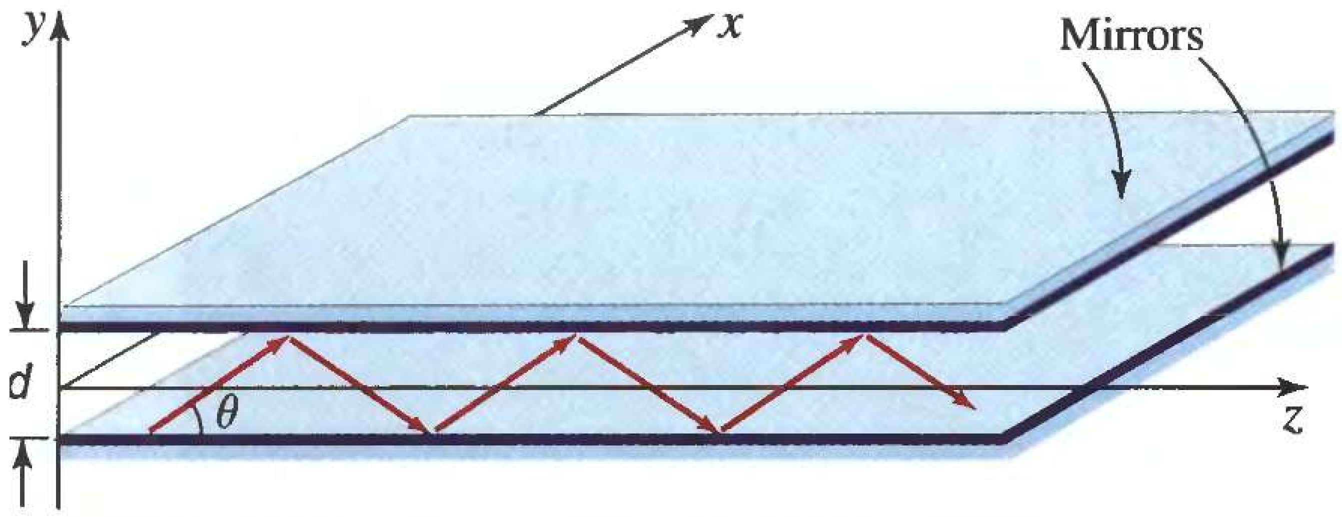


- What if $\sin\theta_2 \geq 1$?
  - There is critical angle $\theta_c = \arcsin n_2/n_1$!
    - $n_1 > n_2$
    - Light prefers to stay in higher index material!
    - Almost everything is reflected → **total internal reflection!**



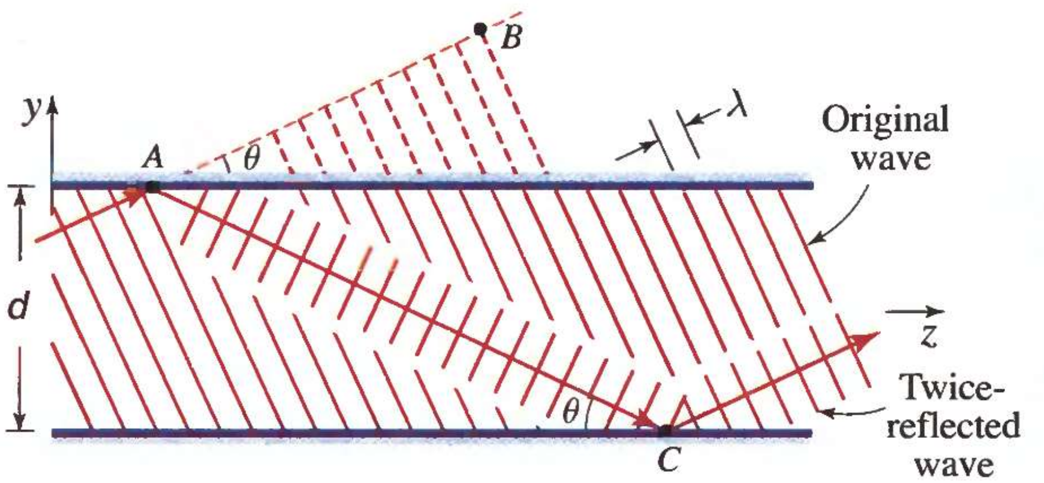
Waveguiding: Theory

- EM field in between two perfect metallic mirrors
  - After each reflection, there is π phase shift



Waveguiding: Theory

- EM field in between two perfect mirrors
  - Interference after second reflection!
  - Self consistency: after second reflection, wave duplicates itself



Definition: EM fields which satisfy this condition, we call (eigen)modes!

Waveguiding: Theory

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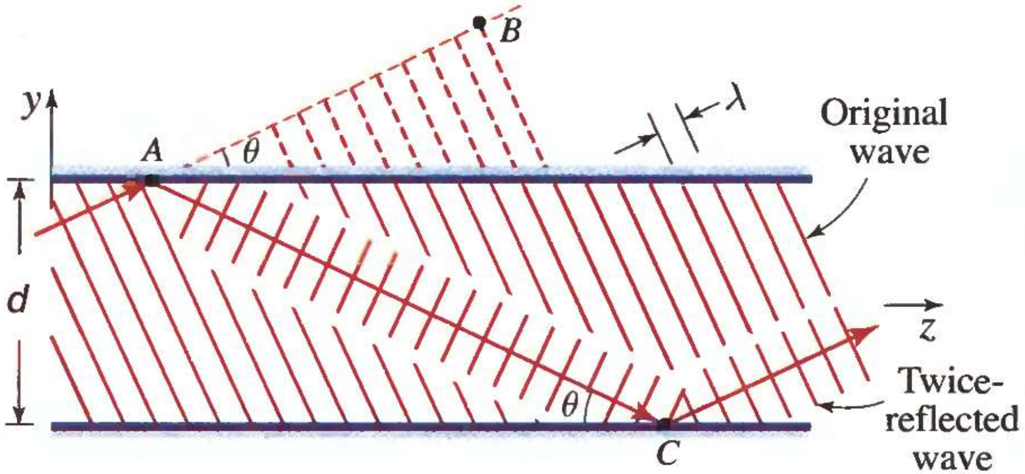

$$\frac{2\pi}{\lambda}\left(\overline{AC}-\overline{AB}\right)=2\pi\cdot(q+1)$$

Definition $k=\frac{2\pi}{\lambda}$

$$k_1\cdot 2d\sin\theta_m=2\pi\cdot m$$

$$m=1,2\ldots$$

Each field bouncing back and forth at angle θ_m is called m^{th} mode!

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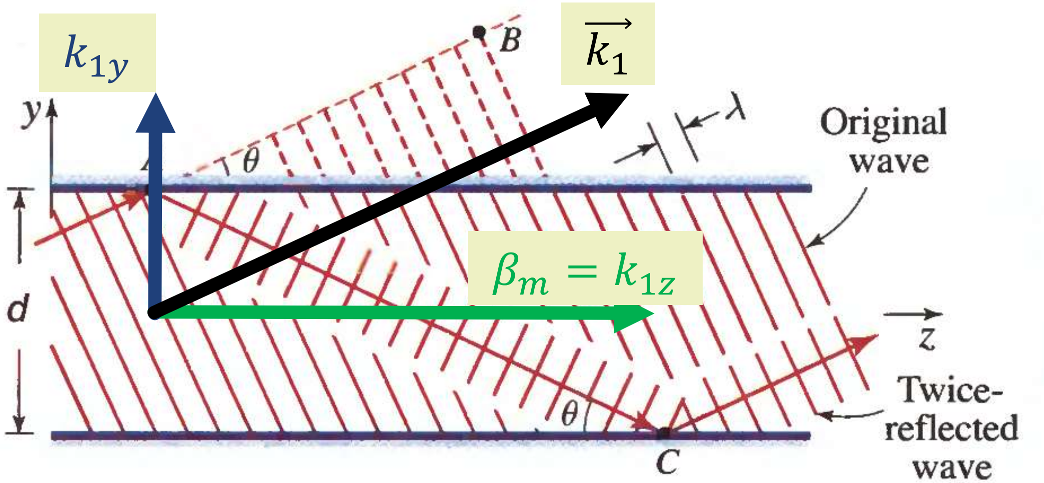

$$k_{1y} = k_1 \sin\theta_m$$

$$k_{1z} = k_1 \cos\theta_m$$

with $k_1 = k_0 * n_1$

Propagation constant: $\beta_m = k_1 \cos\theta_m$

$$\beta_m = k_1 \cos\theta_m = \frac{2\pi}{\lambda} n_{eff,m}$$

Waveguiding: Theory

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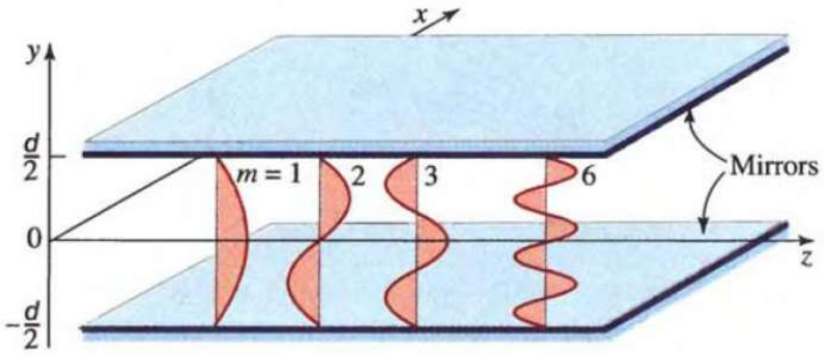

$$2d\cdot k_1\sin\theta_m=2d\cdot k_{1y}=2\pi m$$

$$k_{1y}=\frac{\pi m}{d}\Leftrightarrow\sin\theta_m=\frac{\pi m}{dk_1}<1$$

Number of modes: $M<\frac{2d}{\lambda}n$

Cut-off frequency: $\omega_c\geq\pi\frac{c/n_1}{d}$

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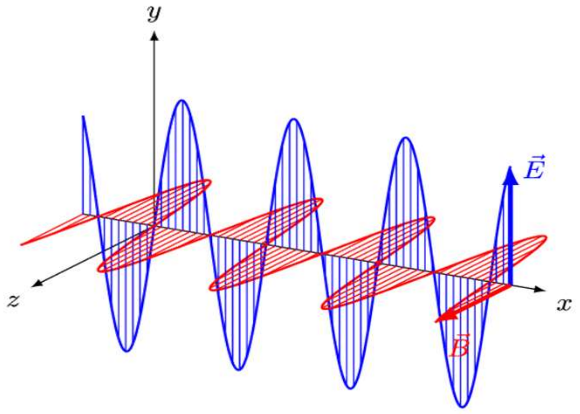
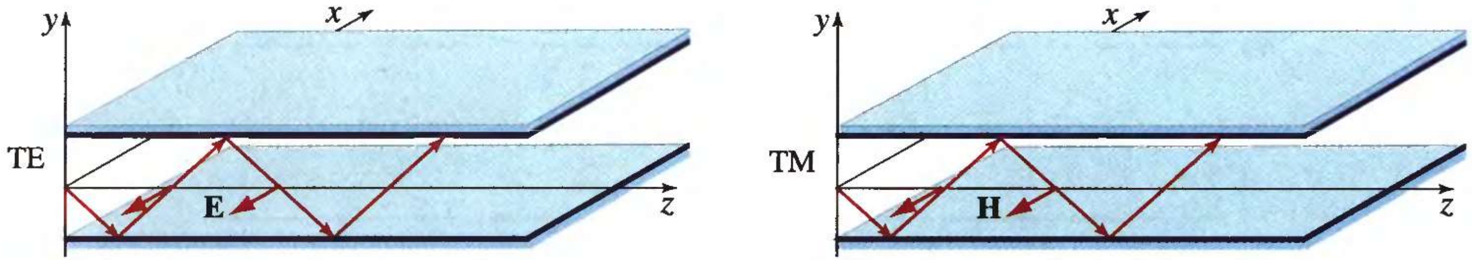
Waveguiding: Theory

- Planar dielectric waveguide
  - Core $(n_1, n_1 > n_2)$
  - Cladding (n_2)
- TE and TM modes





TE mode : *no electric field* in the propagation direction
TM mode: *no magnetic field* in the propagation direction

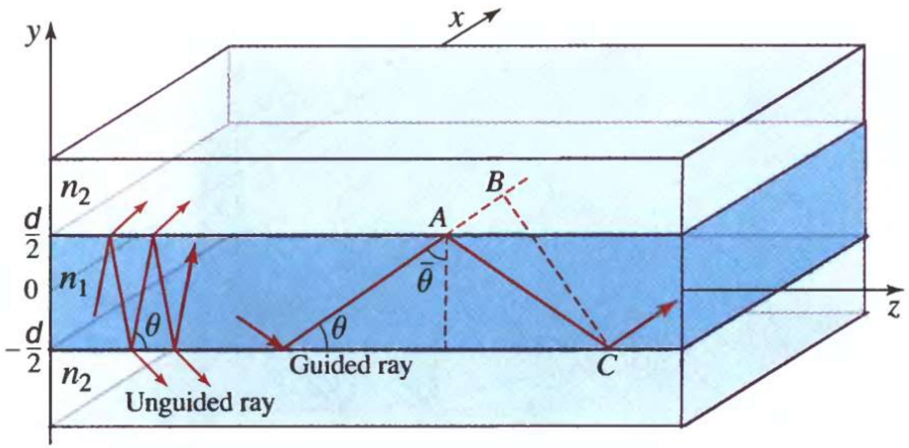
Waveguiding: Theory

- Planar dielectric waveguide
  - Core $(n_1, n_1 > n_2)$
  - Cladding (n_2)

Snell's law:

Ray should be reflected, so there is critical angle for $\bar{\theta} > \bar{\theta}_c = \arcsin\frac{n_2}{n_1}$

This means that $\theta < \theta_c = \frac{\pi}{2} - \overline{\theta_c}$



Waveguiding: Theory

- Planar dielectric waveguide
  - Core $(n_1, n_1 > n_2)$
  - Cladding (n_2)
  - Phase shift after reflection $\neq \pi$

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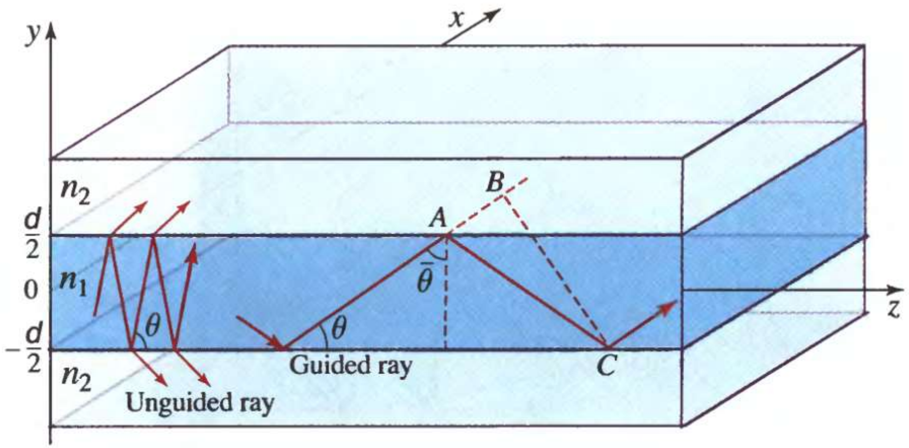
$$\frac{2\pi}{\lambda} 2d \sin\theta_m - 2\phi_r = 2\pi \cdot m$$



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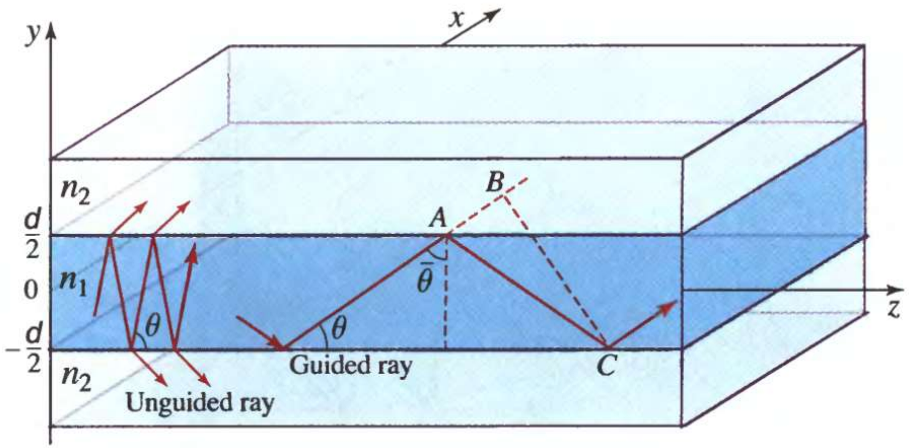

$$\frac{2\pi}{\lambda} 2d \sin\theta_m - 2\phi_r = 2\pi \cdot m$$

Phase change upon reflection

$$k_z = \beta = \frac{2\pi}{\lambda}\cos\theta$$

Waveguiding: Theory

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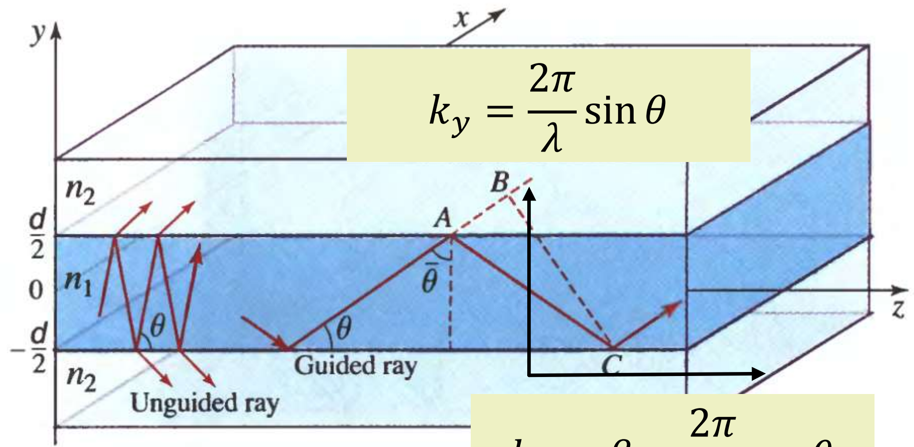

$$k_y = \frac{2\pi}{\lambda} \sin \theta$$

$$k_z = \beta = \frac{2\pi}{\lambda} \cos \theta$$

$$\frac{2\pi}{\lambda} 2d \sin \theta_m - 2\phi_r = 2\pi \cdot m$$

$$\tan \frac{\phi_r}{2} = \begin{cases} \sqrt{\frac{\sin^2 \theta_c}{\sin^2 \theta} - 1}, \textbf{TE} \\ \frac{-1}{\cos^2 \theta_c} \sqrt{\frac{\sin^2 \theta_c}{\sin^2 \theta} - 1}, \textbf{TM} \end{cases}$$

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Waveguiding: Theory

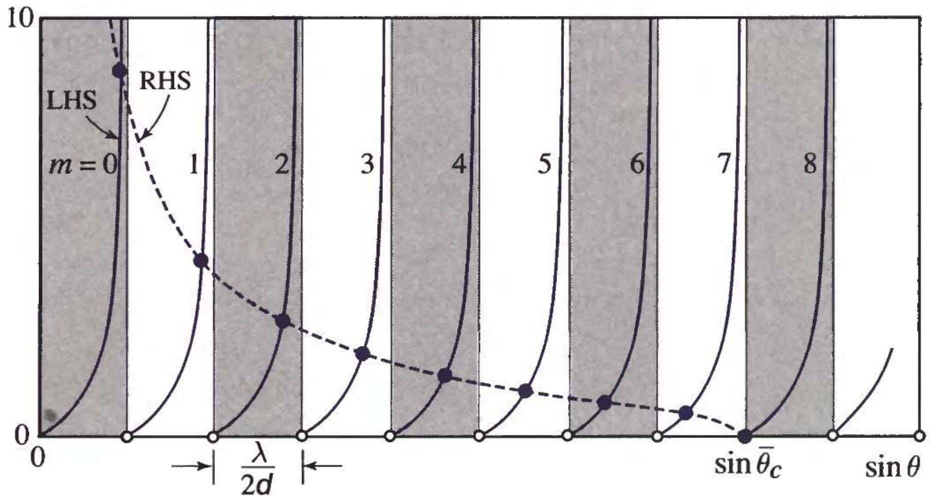
- Planar dielectric waveguide
  - Core $(n_1, n_1 > n_2)$
  - Cladding (n_2)
  - Phase shift after reflection $\neq \pi$
  - Modes can be obtained graphically

$$\tan\left(\frac{\pi d}{\lambda}\sin\theta - m\frac{\pi}{2}\right) = \sqrt{\frac{\sin^2\theta_c}{\sin^2\theta} - 1}$$

Ray should be reflected, so there is critical angle for $\bar{\theta} > \bar{\theta}_c = \arcsin\frac{n_2}{n_1}$

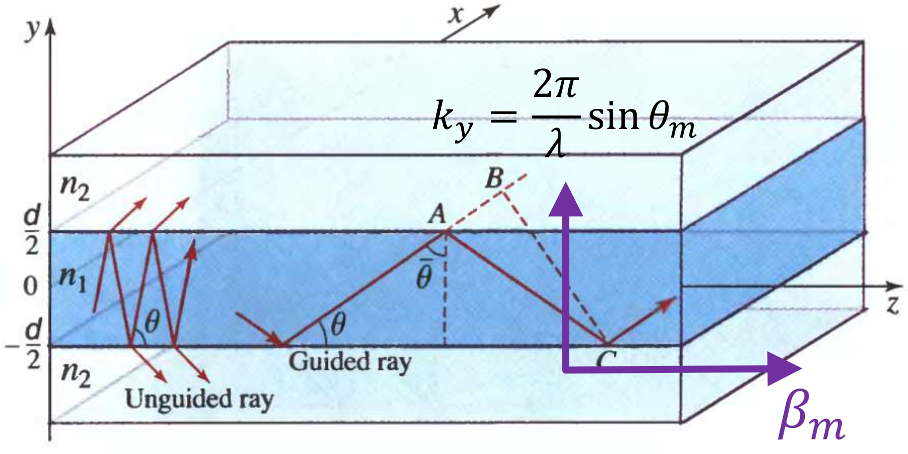
This means that $\theta < \theta_c = \frac{\pi}{2} - \overline{\theta_c}$

$$\frac{2\pi}{\lambda}2d\sin\theta_m - 2\phi_r = 2\pi \cdot m$$



Waveguiding: Theory

- Planar dielectric waveguide
  - Core ($n_1, n_1 > n_2$)
  - Cladding (n_2)
  - Phase shift after reflection $\neq \pi$



$$\beta_m = k_1 \cos\theta_m = k_0 n_1 \cos\theta_m$$

$$n_1 k_0 \geq \beta_m \geq n_2 k_0$$

Snell's law

Waveguiding: Theory

- Planar dielectric waveguide
  - Core ($n_1, n_1 > n_2$)
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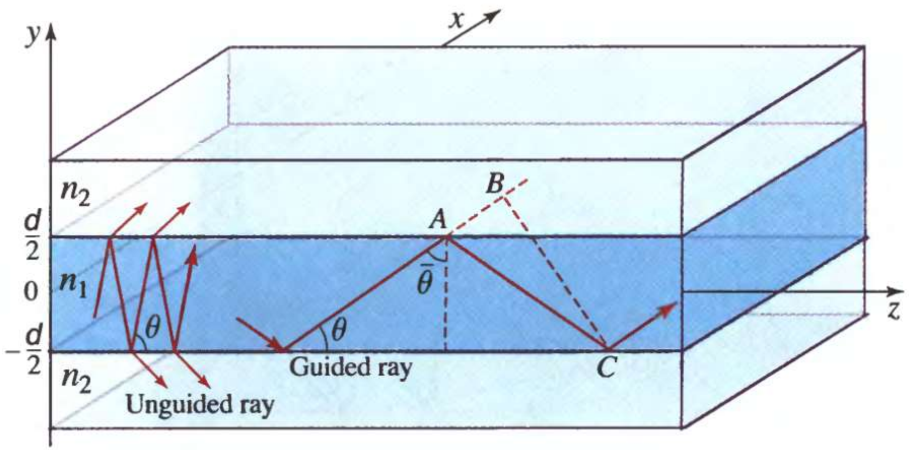
  - Phase shift after reflection $\neq \pi$



$$\frac{\pi m}{d} = k_1 \sin\theta_m \leq k_1 \sin\theta_c$$

Number of modes $M = \frac{1}{\pi} k_1 d \sin(\theta_c) = [\frac{2d}{\lambda_0}\sqrt{n_1^2 - n_2^2}]$

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Waveguiding: Theory

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  - Core ($n_1, n_1 > n_2$)
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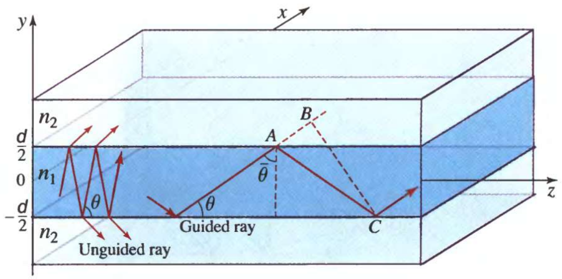
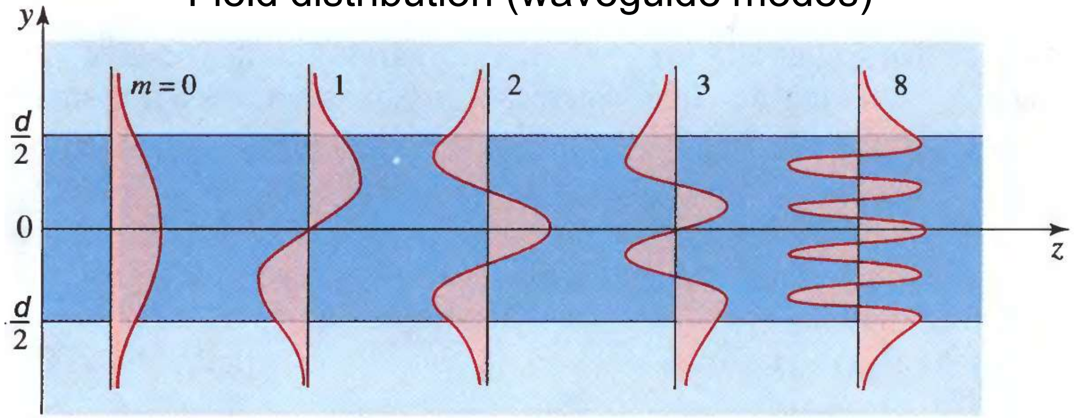
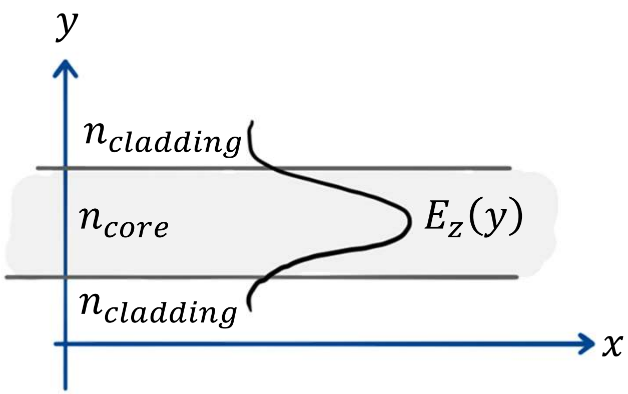


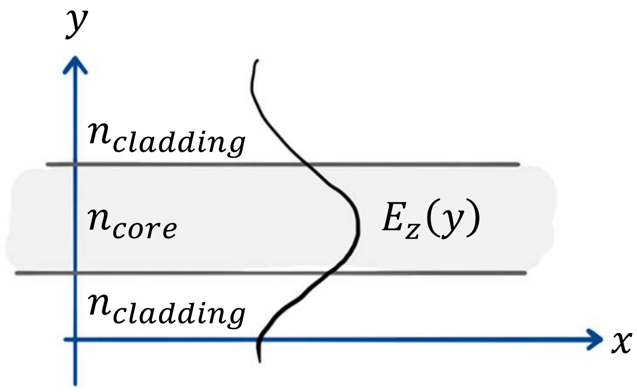
Field distribution (waveguide modes)



Waveguiding: Theory



Strongly confined
Wave mostly in the core



Weakly confined
Wave leaks into cladding

- The confinement depends on n

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