Anti-Reflective Coating with Multiple Layers

Introduction

An anti-reflective coating is a set of thin, transparent films applied to the surface of an optical device such as a lens to reduce reflection. This reduction of reflected light leads to an increase in the efficiency of the optical system and minimizes stray light, which is important in many imaging applications. Anti-reflective coatings can be applied to the surfaces of eyeglasses to reduce glare and make the eyes of the wearer more visible.

The simplest example of an anti-reflective coating is a quarter-wavelength layer, a single dielectric film with thickness equal to one quarter of the wavelength of the incident light. This layer can reduce the reflection coefficient to zero if the refractive index of the film is equal to the geometric mean of the refractive indices of the air ($n_0$) and substrate ($n_S$). For air (1.0) and common glass substrate (1.5) this optimal refractive index would be $\sqrt{(1.0)(1.5)}$ or approximately 1.22.

Typically no material exists with a refractive index that yields a reflection coefficient of exactly zero. Another drawback of the quarter-wavelength layer is that while it can prevent reflection of light at one frequency, it reflects a substantial amount of radiation at any other frequency.

An alternative is to use a coating that consists of multiple layers. Compared to single-layer coatings, a multi-layer coating is more likely to reduce the reflection coefficient across a band of wavelengths and can be produced using a wider variety of real materials.

Model Definition

Figure 1 shows the simple geometry used in this model. It consists of a box with an internal boundary separating the air and substrate domains. This boundary is also where the Thin Dielectric Film features are added.

The red arrow (activated in the Material Discontinuity feature) shows the sense in which the thin dielectric layers are stacked. The last Thin Dielectric Film in the Model Builder represents the topmost thin film in the model.
Simple anti-reflective coatings are designed to minimize the reflectance at and around a specified vacuum wavelength, $\lambda_0$. More sophisticated coatings can minimize reflectance across a relatively large band of wavelengths by employing multiple thin films with different material properties.

The simplest multilayer coating consists of two layers of differing refractive index applied to a glass substrate. Each layer has an optical thickness set to $1/4$ that of the specified vacuum wavelength; therefore this design is called a quarter-quarter coating.

Theoretically, a quarter-quarter coating can reduce the reflectance to zero at the specified wavelength; however, because real materials with ideal refractive indices are seldom available, zero reflectance is normally not achieved.

If the substrate consists of glass ($n_s = 1.5$) the first layer is made of magnesium fluoride, MgF$_2$ ($n_1 = 1.38$) the following expression can be used to determine the optimal refractive index for the second layer, $n_2$, so that the reflectance can be reduced to zero:

$$n_2 = \sqrt{\frac{n_1 n_s}{n_a}}$$
Using this expression the optimal value for \( n_2 \) is 1.69. From Table 1, cerium fluoride, \( \text{CeF}_3 \), has a refractive index close to this value, 1.63. Using this material a minimum reflectance of less that 1\% can be achieved.

**TABLE 1: REFRACTIVE INDICES OF MATERIALS FREQUENTLY USED IN THIN FILMS**

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>REFRACTIVE INDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnesium Flouride (MgF(_2))</td>
<td>1.38</td>
</tr>
<tr>
<td>Silicon Dioxide (SiO(_2))</td>
<td>1.46</td>
</tr>
<tr>
<td>Cerium Fluoride (CeF(_3))</td>
<td>1.63</td>
</tr>
<tr>
<td>Zirconium Oxide (ZrO(_2))</td>
<td>2.2</td>
</tr>
<tr>
<td>Silicon (Si)</td>
<td>3.5</td>
</tr>
</tbody>
</table>

The reflectance as a function of vacuum wavelength is shown in Figure 2. A noticeable drawback of the two-layer coating is that the reflectance is only significantly reduced in a narrow band around a single wavelength.

**Figure 2: Reflectance of a quarter-quarter coating.**

The reflectance can be reduced over a wider range of wavelengths by using a dielectric film with three or more layers. An example of a three-layer coating is the
quarter-half-quarter coating, in which a thin layer of optical thickness $\lambda/2$ is placed between the two quarter-wavelength layers.

**Results and Discussion**

**Figure 3** compares the reflectance of the quarter-quarter and quarter-half-quarter films. Because the refractive indices of real materials are used, the reflectance of the quarter-quarter coating does not decrease to zero. The quarter-half-quarter film exhibits slightly greater reflectance at the center of the band, but the reflectance is reduced over a much wider frequency range.

**Figure 3: Reflectance response for a quarter-quarter and quarter-half-quarter coating configurations.**

**Model Library path:** Ray_Optics_Module/Tutorial_Models/antireflective_coating_multilayer
**Modeling Instructions**

From the **File** menu, choose **New**.

**NEW**
1 In the **New** window, click **Model Wizard**.

**MODEL WIZARD**
1 In the **Model Wizard** window, click **2D**.
2 In the **Select physics** tree, select **Optics>Ray Optics>Geometrical Optics (gop)**.
3 Click **Add**.
4 Click **Study**.
5 In the **Select study** tree, select **Preset Studies>Ray Tracing**.
6 Click **Done**.

**DEFINITIONS**

**Parameters**
1 On the **Model** toolbar, click **Parameters**.
2 In the **Settings** window for Parameters, locate the **Parameters** section.
3 In the table, enter the following settings:

<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>n_air</td>
<td>1</td>
<td>1.0000</td>
<td>Refractive index for air</td>
</tr>
<tr>
<td>n_glass</td>
<td>1.5</td>
<td>1.5000</td>
<td>Refractive index for glass</td>
</tr>
<tr>
<td>n_CeF3</td>
<td>1.63</td>
<td>1.6300</td>
<td>Refractive index for CeF3</td>
</tr>
<tr>
<td>n_MgF2</td>
<td>1.38</td>
<td>1.3800</td>
<td>Refractive index for MgF2</td>
</tr>
<tr>
<td>n_ZrO2</td>
<td>2.2</td>
<td>2.2000</td>
<td>Refractive index for ZrO2</td>
</tr>
<tr>
<td>lam0</td>
<td>550[nm]</td>
<td>5.5000E-7 m</td>
<td>Vacuum wavelength</td>
</tr>
</tbody>
</table>

**GEOMETRY 1**

**Square 1 (sq1)**
1 On the **Geometry** toolbar, click **Primitives** and choose **Square**.
2 In the **Settings** window for Square, click to expand the **Layers** section.
In the table, enter the following settings:

<table>
<thead>
<tr>
<th>Layer name</th>
<th>Thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

4 Click the Build All Objects button.

GEOMETRICAL OPTICS (GOP)

1 In the Model Builder window, under Component 1 (comp1) click Geometrical Optics (gop).
2 In the Settings window for Geometrical Optics, locate the Ray Properties section.
3 From the Intensity computation list, choose Using principal curvatures and ray power.
4 Locate the Advanced Settings section. Select the Frequency-dependent refractive indices check box.
5 In the Maximum number of secondary rays text field, type 0.

Set up the air domain.

Medium Properties 1

1 In the Model Builder window, under Component 1 (comp1)>Geometrical Optics (gop) click Medium Properties 1.
2 In the Settings window for Medium Properties, locate the Medium Properties section.
3 In the n text field, type n_air.
4 On the Physics toolbar, click Domains and choose Medium Properties.

Set up the glass substrate.

Medium Properties 2

1 Select Domain 1 only.
2 In the Settings window for Medium Properties, locate the Medium Properties section.
3 In the n text field, type n_glass.

Set up the quarter-quarter anti-reflective coating using two Thin Dielectric Film features.

It is useful to display the boundary normal in the Graphics window since this indicates the order in which the thin films are arranged. The arrow points in the direction away from the substrate, so the first layer that appears in the Model Builder is adjacent to the substrate and the last layer that appears is adjacent to the air domain.
Material Discontinuity 1
1 In the Model Builder window, under Component 1 (comp1)>Geometrical Optics (gop)
click Material Discontinuity 1.
2 In the Settings window for Material Discontinuity, locate the Advanced Settings
section.
3 Select the Show boundary normal check box.
4 On the Physics toolbar, click Attributes and choose Thin Dielectric Film.
The first layer (directly touching the glass substrate) is the CeF$_3$ layer with refractive
index of 1.63. The thickness is set to be a quarter of the wavelength in this material.

Thin Dielectric Film 1
1 In the Settings window for Thin Dielectric Film, locate the Film Properties section.
2 In the $n$ text field, type $n_{CeF3}$.
3 In the $t$ text field, type $\lambda_{0}/(4*n_{CeF3})$.

Material Discontinuity 1
1 On the Physics toolbar, click Attributes and choose Thin Dielectric Film.
The second layer (on top of layer 1) is the CeF$_2$ layer with refractive index of 1.38. The
thickness of this layer is also set to a quarter of the wavelength.

Thin Dielectric Film 2
1 In the Settings window for Thin Dielectric Film, locate the Film Properties section.
2 In the $n$ text field, type $n_{MgF2}$.
3 In the $t$ text field, type $\lambda_{0}/(4*n_{MgF2})$.

Next, set up the Release From Grid feature to release a number of rays of different
wavelengths from 400 nm to 800 nm.

Release from Grid 1
1 On the Physics toolbar, click Global and choose Release from Grid.
2 In the Settings window for Release from Grid, locate the Initial Coordinates section.
3 In the $q_{x,0}$ text field, type 0.5.
4 In the $q_{y,0}$ text field, type 1.
5 Locate the Ray Direction Vector section. Specify the $L_{0}$ vector as

\begin{array}{c|c}
0 & x \\
-1 & y \\
\end{array}
Locate the Initial Ray Frequency section. From the Distribution function list, choose List of values.

Click Range.

In the Range dialog box, choose Number of values from the Entry method list.

In the Start text field, type $3 \times 10^8 \text{m/s}/400\text{nm}$.

In the Stop text field, type $3 \times 10^8 \text{m/s}/800\text{nm}$.

In the Number of values text field, type 100.

Click Replace.

STUDY 1

Set up a Ray Tracing study step to compute the ray trajectories to a maximum optical path length of 1 m.

Step 1: Ray Tracing

1 In the Model Builder window, expand the Study 1 node, then click Step 1: Ray Tracing.
2 In the Settings window for Ray Tracing, locate the Study Settings section.
3 From the Time step specification list, choose Specify maximum path length.
4 In the Lengths text field, type range(0,0.01,1.1).
5 From the Stop condition list, choose No active rays remaining.
6 On the Model toolbar, click Compute.

RESULTS

1D Plot Group 2

1 On the Model toolbar, click Add Plot Group and choose 1D Plot Group.
2 In the Settings window for 1D Plot Group, locate the Data section.
3 From the Data set list, choose Ray 1.
4 From the Time selection list, choose Last.
5 Locate the Plot Settings section. Select the x-axis label check box.
6 In the associated text field, type Vacuum wavelength (nm).
7 Select the y-axis label check box.
8 In the associated text field, type Reflectance (%).
9 On the 1D plot group toolbar, click Ray.
Plot the percentage reflectance.
In the **Settings** window for Ray, locate the **y-Axis Data** section.

In the **Expression** text field, type $100 \times (\text{gop}.\text{relg1.Q0}-\text{gop}.\text{Q})/\text{gop}.\text{relg1.Q0}$.

Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

In the **Expression** text field, type $\text{gop}.\lambda_0$.

From the **Unit** list, choose **nm**.

Click to expand the **Legends** section. Select the **Show legends** check box.

From the **Legends** list, choose **Manual**.

In the table, enter the following settings:

<table>
<thead>
<tr>
<th>Legends</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter-Quarter</td>
</tr>
</tbody>
</table>

On the **1D plot group** toolbar, click **Plot**.

To model a quarter-half-quarter layer, add another **Thin Dielectric Film** feature. The material of the middle layer is chosen to be Zirconium Oxide, ZrO$_2$, with refractive index 2.2. Set the thickness to half the specified wavelength.

**GEOMETRICAL OPTICS (GOP)**

*Thin Dielectric Film 3*

1. On the **Physics** toolbar, click **Attributes** and choose **Thin Dielectric Film**.

This layer will sit between the other two thin layers so its node must be moved in the Model Builder.

2. In the **Model Builder** window, under **Component 1 (comp1)>Geometrical Optics (gop)>Material Discontinuity 1** right-click **Thin Dielectric Film 3** and choose **Move Up**.

3. In the **Settings** window for Thin Dielectric Film, locate the **Film Properties** section.

4. In the $n$ text field, type $n_{\text{ZrO2}}$.

5. In the $t$ text field, type $\lambda_0/(2*n_{\text{ZrO2}})$.

Add another study so that the two films can be compared.

**ADD STUDY**

1. On the **Model** toolbar, click **Add Study** to open the **Add Study** window.

2. Go to the **Add Study** window.

3. Find the **Studies** subsection. In the **Select study** tree, select **Preset Studies>Ray Tracing**.
4 Click **Add Study** in the window toolbar.

**STUDY 2**

*Step 1: Ray Tracing*

1 On the **Model** toolbar, click **Add Study** to close the **Add Study** window.
2 In the **Model Builder** window, under **Study 2** click **Step 1: Ray Tracing**.
3 In the **Settings** window for Ray Tracing, locate the **Study Settings** section.
4 From the **Time step specification** list, choose **Specify maximum path length**.
5 In the **Lengths** text field, type **range(0,0.01,1.1)**.
6 From the **Stop condition** list, choose **No active rays remaining**.
7 On the **Model** toolbar, click **Compute**.

**RESULTS**

*1D Plot Group 2*

1 In the **Model Builder** window, under **Results>1D Plot Group 2** right-click **Ray 1** and choose **Duplicate**.
2 In the **Settings** window for Ray, locate the **Data** section.
3 From the **Data set** list, choose **Ray 2**.
4 From the **Time selection** list, choose **Last**.
5 Click to expand the **Legends** section. In the table, enter the following settings:

<table>
<thead>
<tr>
<th>Legends</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter-Half-Quarter</td>
</tr>
</tbody>
</table>

6 On the **1D plot group** toolbar, click **Plot**.

Compare the result with **Figure 3**.